

NUMBER SYSTEM

Introduction

Number System is a method of writing numerals to represent numbers.

- ❖ Ten symbols 0,1,2,3,4,5,6,7,8,9 are used to represent any number (however large it may be) in our number system.
- ❖ Each other symbols 0,1,2,3,4,5,6,7,8,9 is called a digit or a figure.

Integers

The set of integers is the set of natural number, Zero and negative of natural number simultaneously. The set of integers is denoted by I or Z

- ❖ $Z = \{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$

Natural Number

- ❖ Positive integers 1,2,3,4... are called Natural numbers.
- ❖ Counting number 1,2,3,4,5... are called natural numbers.
- ❖ The set of natural number is denoted by N i.e., $N = \{1, 2, 3, 4, 5, \dots\}$

Whole Number

- ❖ All natural number together with zero are called whole numbers, as 0,1,2,3,4... are whole numbers.
- ❖ The set of whole number is denoted by W , i.e. $W = \{1, 2, 3, 4, 5, \dots\}$ So, $W = N \cup \{0\}$, where N is the set of natural numbers..
- ❖ 0 is the smallest whole number, there is no largest whole number i.e. the number of the number of the elements in the set of whole numbers is infinite.
- ❖ Every natural number is a whole number i.e. $N \subset W$ i.e. N is a subset of W .
- ❖ 0 is a whole number, but not a natural number i.e., $0 \in W$ but $0 \notin N$
- ❖ N is also a proper subset of W , i.e. $N \subset W$.

Even Numbers

- ❖ Whole numbers which are exactly divisible by 2 are called even numbers.
- ❖ The set of even numbers is denoted by 'E', such that $E = \{0, 2, 4, 6, 8, \dots\}$.

Odd Numbers

- ❖ Natural numbers which are not exactly divisible by 2 are called Odd numbers. $O = \{1, 3, 5, 7, 9, \dots\}$.

Prime Numbers

Natural numbers having exactly two distinct factors i.e. 1 and the number itself are called Prime numbers.

- ❖ 2 is the smallest and only even prime number.

Identification of Prime Number

- Step (i) :** Find approximate square root of given number
- Step (ii) :** Divide the given number by prime numbers less than approximately square root of number. If given number is not number is prime otherwise not.

Ex.1 Is 131 a prime number ?

Sol. Approximate square root = 12

Prime number < 12 are 2,3,5,7,11. But 131 is not divisible by any of these prime number. So 131 is a prime number.

Composite Numbers

Natural numbers having more than two factors are called Composite numbers.

- ❖ 4,6,8,9,10,12,14,15,16,18... are composite numbers.
- ❖ Number 1 is neither prime nor composite number.
- ❖ All even number except 2 are composite numbers.
- ❖ Every natural number except 1 is either prime or composite number.
- ❖ There are infinite prime numbers and infinite composite numbers.

Co- Prime Number or Relatively Prime Numbers

- ❖ Two natural numbers are said to be co-prime numbers or relatively prime numbers if they have only 1 as common factor. For ex. 8,9; 15,16; 26,33 etc. are co-prime numbers.
- ❖ Co-prime numbers may not themselves be prime numbers. As 8 and 9 are co-prime numbers, but neither 8 nor 9 is a prime number.
- ❖ Every two consecutive natural number are co-primes.

Twin primes

- ❖ Pairs of prime numbers which have only one composite number between them called Twin primes. 3,5; 5,7; 11,13; 17,19; 29,31; 41,43; 59,61 and 71,73 etc. are twin primes.

Divisibility Test

Number	Divisibility
2	Unit digit should be 0 or even
3	The sum of digits of no. should be divisible by 3
4	The no. formed by last 2 digits of given no. should be divisible by 4.
5	Unit digit should be 0 or 5
6	No. should be divisible by 2 & 3 both
8	The number formed by last 3 digits of given no. should be divisible by 8.
9	Sum of digits of given no should be divisible by 9
11	The difference between sums of the digits at even & at odd places should be zero or multiple of 11.
25	Last 2 digits of the number should be 00,25,50 or 75

Ex.2 Which one of the following number is exactly divisible by 11 ?

- (A) 235641 (B) 245642
(C) 315624 (D) 415624

Sol. (A) $(1+6+3) - (2+5+4) = 1$ No
(B) $(2+6+4) - (2+5+2) = 0$ Yes
(C) $(4+6+1) - (2+5+3) = 1$ No
(D) $(4+6+1) - (2+5+4) = 0$ (Yes).

Rational Numbers

These are real numbers which can be expressed in the form of p/q where p and q are integers and $q \neq 0$ e.g. $2/3, 37/15, -17/19$.

- ❖ All natural numbers, whole numbers and integers are rational.
- ❖ Rational numbers include all integers and (without any decimal part to it), terminating fractions (fractions in which the decimal parts terminating e.g. $0.75, -0.02$ etc.) and also non-terminating but recurring decimals e.g. $0.666\dots, -2.333\dots$, etc.

Fractions

- (a) Common fraction : Fractions whose denominator is not 10.
(b) Decimal fraction : Fractions whose denominator is 10 or any power of 10.
(c) Proper fraction :

Numerator < Denominator i.e. $\frac{3}{5}$.

- (d) Improper fraction :

Numerator > Denominator i.e. $\frac{5}{3}$.

- (e) Mixed fraction :

Consists of integral as well as fractional part i.e. $3\frac{2}{7}$.

- (f) Compound fraction : Fraction whose numerator and denominator themselves are fractions. i.e. $\frac{2/3}{5/7}$

❖ Improper fraction can be written in the form of mixed fraction.

Irrational Numbers

All real number which are not rational are irrational numbers. These are non-recurring as well as non-terminating type of decimal numbers

e.g. $\sqrt{2}, \sqrt[3]{4}, 2 + \sqrt{3}, \sqrt{2 + \sqrt{3}}, \sqrt[4]{7\sqrt{3}}$ etc.

Pure Recurring Decimal

If in a decimal fraction, a figure or a set of figures is repeated continuously, then such a number is called recurring decimal.

Thus, $\frac{1}{3} = 0.333\dots = 0.\overline{3}$

$\frac{22}{7} = 3.142857142857\dots = 3.\overline{142857}$

Conversion of decimal numbers into rational numbers of the form p/q .

Short method for pure recurring decimal : Write the repeated digit or digits only once in the numerator and take as many nines in the denominator as there are repeating digits in the given number.

e.g. (i) $0.\overline{3} = 3/9$ or $1/3$

(ii) $0.\overline{387} = 387/999$

Mixed Recurring Decimal

A decimal is said to be a mixed recurring decimal if there is at least one digit after the decimal point, which is not repeated.

Short cut method for mixed recurring decimal : Form a fraction in which numerator is the difference between the number formed by all the digits after the decimal point taking the repeated digits only once and that formed by the number formed by as many nines as there are repeated digits followed by as many zeros as the number of non-repeated digits.

Ex.3 Change $0.74\overline{35}$ in the form of p/q .

Sol. $0.74\overline{35} = \frac{7435 - 74}{9900} = \frac{7361}{9900}$

Comparison Fractions

Suppose some fractions are to be arranged in ascending or descending order of magnitude. Then convert each one of the given fractions in the decimal form, and arrange them accordingly,

Now, $\frac{3}{5} = 0.6, \frac{6}{7} = 0.857, \frac{7}{9} = 0.777\dots$

Since $0.857 > 0.777\dots > 0.6$, so $\frac{6}{7} > \frac{7}{9} > \frac{3}{5}$

Real Numbers

- ❖ The sets of rational numbers and irrational numbers taken together are known as a set of real numbers.

Modulus of a Real Numbers

The absolute value of a real number $|x|$ is defined as

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

For example :

$$|2| = 2; 2 > 0 \text{ and } |-2| = -(-2) = 2; -2 < 0.$$

Bodmas Rule

This rule depicts the correct sequence in which the operations are to be executed so as to find the value of a given expression.

Here 'B' stands for 'Bracket', 'O' for 'D' for Division 'M' for Multiplication, 'A' for 'Addition' and 'S' for subtraction.

Thus, in simplifying an expression, first of all the brackets must be removed, strictly in the order $()$, $\{\}$ and $[\]$.

- (i) Of (ii) Division (iii) Multiplication
(iv) Addition (v) Subtraction.

Vinculum (or Bar) : When an expression contains Vinculum, before applying under the Vinculum.

Ex.4 Simplify : $3\frac{1}{3} \div \left\{ 1\frac{1}{2} - \frac{1}{2} \left(2\frac{1}{2} - \frac{1}{4} - \frac{1}{6} \right) \right\}$

Sol. Given exp. $\left[\frac{13}{4} \div \left\{ \frac{5}{4} - \frac{1}{2} \left(\frac{5}{2} - \frac{3}{12} - \frac{2}{12} \right) \right\} \right]$

$$= \left[\frac{13}{4} \div \left\{ \frac{5}{4} - \frac{1}{2} \left(\frac{5}{2} - \frac{1}{12} \right) \right\} \right] = \left[\frac{13}{4} \div \left\{ \frac{5}{4} - \frac{1}{2} \left(\frac{30-1}{12} \right) \right\} \right]$$

$$= \left[\frac{13}{4} \div \left\{ \frac{5}{4} - \frac{29}{24} \right\} \right] = \left[\frac{13}{4} \div \left\{ \frac{30-29}{24} \right\} \right]$$

$$= \left[\frac{13}{4} \div \frac{1}{24} \right] = \left[\frac{13}{4} \times 24 \right] = 78.$$

Square and Square Roots

Squares : When a number is multiplied by itself then the product is called the square of that number.

Perfect Square : A natural number is called a perfect square if it is the square of any other natural number e.g. 1,4,9,... are the squares of 1,2,2,... respectively.

- Ex. 5** Find the least perfect square which is exactly divisible by each of the numbers 6,9,15 and 20.

Sol

3	6,9,15,20
5	2,3,5,20
2	2,3,1,4
	1,3,1,2

$$\therefore \text{LCM} = 3 \times 5 \times 2 \times 3 \times 2 = 180$$

The least multiple of 180 which is a perfect square is $18 \times 5 = 900$.

Square roots : The square root of a number x is that number which when multiplied by itself gives x as the product. As we say square of 3 is 9, then we can also say that square root of 9 is 3.

The symbol used to indicate the square root of a number is ' $\sqrt{\quad}$ ', i.e. $\sqrt{81} = 9$, $\sqrt{225} = 15$ etc.

- ❖ We can calculate the square root of positive numbers only. However the square root of a positive number may be a positive or a negative number.

e.g. $\sqrt{25} = +5$ or -5

Properties of Square Roots :

- If the unit digit of a number is 2,3,7 or 8, then it does not have a square root in \mathbb{N}
- If a number ends in an odd number of zeros, then it does not have a square root in \mathbb{N}
- The square root of an even number is even and square root of an odd number is odd.

e.g. $\sqrt{81} = 9$, $\sqrt{256} = 16$, $\sqrt{324} = 18$...etc

- Negative numbers have no square root in set of real numbers.

Ex.6 if $\sqrt{18225} = 135$, then find the value of

$$(\sqrt{182.25} + \sqrt{1.8225} + \sqrt{0.018225} + \sqrt{0.00018225})$$

Sol. Given exp.

$$= \frac{\sqrt{18225}}{10^2} + \sqrt{\frac{18225}{10^4}} + \sqrt{\frac{18225}{10^6}} + \sqrt{\frac{18225}{10^8}}$$

$$= \frac{\sqrt{18225}}{10} + \frac{\sqrt{18225}}{10^2} + \frac{\sqrt{18225}}{10^3} + \frac{\sqrt{18225}}{10^4}$$

$$= \frac{135}{10} + \frac{135}{100} + \frac{135}{1000} + \frac{135}{10000}$$

$$13.5 + 1.35 + 0.135 + 0.0135 = 14.9985.$$

Cube and Cube Roots

Cube : if any number is multiplied by itself three times then the result is called the cube of that number.

Perfect cube : A natural number is said to be a perfect cube if it is the cube of any other natural number.

Cube roots : The cube root of a number x is that number whose cube gives x .

The cube root of x is denoted by symbol $\sqrt[3]{x}$. Thus,

$$\sqrt[3]{8} = 2, \sqrt[3]{27} = 3, \sqrt[3]{64} = 4, \sqrt[3]{125} = 5 \text{ and so on.}$$

- Ex. 7** By what least number 675 be multiplied to obtain a number which is a perfect cube ?

Sol. $675 = 5 \times 5 \times 3 \times 3 \times 3$

To make it a perfect cube, it must be multiplied by 5.

Ex. 8 Find the cube root of .000216.

Sol. $(0.000216)^{1/3} = \left(\frac{216}{10^6}\right)^{1/3} = \left(\frac{6 \times 6 \times 6}{10^2 \times 10^2 \times 10^2}\right)^{1/3}$
 $= \frac{6}{10^2} = \frac{6}{100} = 0.06$

Factors and Multiples

Factors : 'a' is a factor of 'b' if there exists a relation such that $a \times n$, where 'n' is any natural number.

Number of factors : For any composite number C, which can be expressed as $C = a^p \times b^q \times c^r \times \dots$ where a,b,c..... are all prime factors and p,q,r are positive integers, then the number of factors is equal to $(p+1) \times (q+1) \times (r+1) = \text{e.g. } 36 = 2^2 \times 3^2$. So the factors of $36 = (2+1) \times (2+1) = 3 \times 3 = 9$.

Ex.9 Find the total number of factors in the expression $(4)^{11} \times (7)^5 \times (11)^2$

Sol. $(4)^{11} \times (7)^5 \times (11)^2$
 $= (2 \times 2)^{11} \times (7)^5 \times (11)^2$
 $= 2^{22} \times 7^5 \times 11^2$
 \therefore Total number of factors $= (22+1) (5+1) (2+1) = 414$.

Divisibility

Division Algorithm : General representation of result is

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}$$

Ex.10 On dividing 4150 by certain number, the quotient is 55 and the remainder is 25. Find the divisor.

Sol, $4150 = 55 \times x + 25$
 $\Rightarrow 55x = 4125$
 $\Rightarrow x = \frac{4125}{55} = 75$

Remainders

The method of finding the remainder without actually performing the process of division is termed as remainder theorem.

❖ Remainder should always be positive. For example if we divide -22 by 7, generally we -3 as quotient and -1 as remainder. But this is wrong because remainder is never be negative is +6. We can also get remainder 6 by adding -1 to divisor $7 (7-1=6)$.

Ex. 11 A number when divided by 296 gives a remainder 75. When the same number is divided by 37, then find the remainder

Sol. Number $= (296 \times Q) + 75 = (37 \times 8Q) + (37 \times 2) + 1$
 $= 37 \times (8Q + 2) + 1$
 \therefore Required remainder = 1.

Cyclicity

We are having 10 digits in our number systems and some of them shows special characteristics like they, repeat their unit digit after a cycle, for example 1 repeat its unit after every consecutive power. So, its cyclicity is 1 on the other hand digit 2 repeat its unit digit. The cyclicity of digits are as follows

Digit	Cyclicity
0,1,5 and 6	1
4 and 9	2
2,3,7 and 8	3

So, if we want to find the last digit of 2^{45} , divide 45 by 4. The remainder is 1 so the last digit of 2^{45} would be same as the last digit of 2^1 which is 2.

Ex.12 Find the unit digit in the product $(7^{71} \times 6^{59} \times 3^{65})$.

Sol. Unit digit in 7^4 is 1.

\therefore Unit digit in 7^{68} is 1.
 Unit digit in 7^{71} is 3.

[$1 \times 7 \times 7 \times 7$ given unit digit 3]

Again, every power of 6 will give unit digit 6.

\therefore Unit digit in 6^{59} is 6.
 Unit digit in 3^4 is 1.

\therefore Unit digit in 3^{64} is 1. Unit digit in 3^{65} is 3.

\therefore Unit digit in $(7^{71} \times 6^{59} \times 3^{65})$

\Rightarrow Unit digit in $(6 \times 6 \times 3) = 4$.

EXERCISE

- In numbers from 1 to 100 digit "0" appears _____ times.
 (A) 9 (B) 10
 (C) 11 (D) 12
- How many numbers are there containing 2 digits.
 (A) 90 (B) 9
 (C) 100 (D) 89
- $42(4+2) = (42 \times 4) + (42 \times 2)$ is an example of
 (A) Closure property
 (B) Commutative property
 (C) Associative property
 (D) Distributive property
- $38 + 83 = 83 + 38$ is an example of :
 (A) Commutative property
 (B) Associative property
 (C) Closure property
 (E) Distributive property
- Successor of 301999 is _____
 (A) 30200 (B) 302000
 (C) 302010 (D) 301100
- Which of the following statement is true ?
 (A) Every whole number is a natural number
 (B) Every natural number is a whole number
 (C) '1' is the least whole number
 (D) None of these
- What least number should be added to 1330 to get number exactly divisible by 43 ?
 (A) 46 (B) 1
 (C) 3 (D) 7

8. The smallest number which when divided by 20, 25, 35 and 40 and leaves a remainder of 14, 19, 29, and 34 respectively is :

- (A) 1394 (B) 1404
(C) 1664 (D) 1406

9. The least number which when decreased by 7 is exactly divisible by 12, 16, 18, 21 and 28 is :

- (A) 1012 (B) 1008
(C) 1015 (D) 1022

10. The greatest number that will divide 137, 182 and 422 leaving a remainder of 2 in each case is :

- (A) 15 (B) 12
(C) 21 (D) None of these

11. The smallest number which when divided by 4, 6, 10, 15 gives the same remainder 3 is :

- (A) 75 (B) 123
(C) 63 (D) 39

12. Division of $\bar{5}.3716$ by 3 is given by :

- (A) - 2.4572 (B) $\bar{1}.7905$
(C) $\bar{2}.4572$ (D) $\bar{2}.5472$

13. The last digit of the number $(373)^{333}$ is :

- (A) 1 (B) 2
(C) 3 (D) 9

14. The two missing number shown with asterisk in the equation $5\frac{3}{*} \times * \frac{1}{2} = 19$ are :

- (A) 6,3 (B) 7,3
(C) 8,3 (D) 11,3

15. Given $\sqrt{5} = 2.236$ the value of $\sqrt{45} + \sqrt{605} - 245$ correct to 3 decimal places is :

- (A) 15.652 (B) 11.180
(C) 18.652 (D) 16.652

16. A student was asked to multiply a number by $\frac{3}{2}$

instead he divided the number $\frac{3}{2}$ and obtained a number smaller by $\frac{2}{3}$; the number is :

- (A) $\frac{4}{5}$ (B) $\frac{3}{5}$
(C) $\frac{2}{3}$ (D) $\frac{1}{2}$

17. Which of the following statements is true ?

- (A) $\frac{-2}{3} < \frac{4}{-9} < \frac{-5}{12} < \frac{7}{-18}$
(B) $\frac{7}{-18} < \frac{-5}{12} < \frac{4}{-9} < \frac{-2}{3}$
(C) $\frac{4}{-9} < \frac{7}{-18} < \frac{-5}{12} < \frac{-2}{3}$
(D) $\frac{-2}{3} < \frac{-5}{12} < \frac{4}{-9} < \frac{7}{-18}$

18. Which of the following rational numbers lie between $\frac{-3}{7}$ and $\frac{-9}{8}$?

- (A) $\frac{-1}{2}$ (B) 0
(C) $\frac{12}{15}$ (D) None of these

19. 0.018 can be expressed in the rational form as :

- (A) $\frac{18}{1000}$ (B) $\frac{18}{990}$
(C) $\frac{18}{9900}$ (D) $\frac{18}{999}$

20. The least number which must be subtracted from 2509 to make it a perfect square is :

- (A) 6 (B) 9
(C) 12 (D) 14

21. $\sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}$

- (A) equal 1
(B) lies between 0 and 1
(C) lies between 1 and 2
(D) is greater than 2

22. if $x * y = \sqrt{x^2 + y^2}$, the value of $(1 * 2\sqrt{2})(1 * -2\sqrt{2})$ is :

- (A) -7 (B) 0
(C) 2 (D) 9

23. The value of $4 - \frac{5}{1 + \frac{1}{1 + \frac{1}{2\frac{1}{4}}}}$

- (A) $\frac{40}{31}$ (B) $\frac{4}{9}$
(C) $\frac{1}{8}$ (D) $\frac{31}{40}$

24. If A and B are real numbers and $A^2 + B^2 = 0$ then :

- (A) $A > 0, B > 0$ (B) $A < 0, B > 0$
(C) $A > 0 = B$ (D) $A = -B$

25. A two digit number is divisible by 3 and 4. Also the difference between the unit's digit and the ten's digit is equal to 4. The number is :

- (A) 96 (B) 48
(C) 36 (D) 72

26. Choose the rational number high does not lie between rational numbers $-\frac{2}{5}$ and $-\frac{1}{5}$.

- (A) $-\frac{1}{4}$ (B) $-\frac{3}{20}$
(C) $-\frac{3}{10}$ (D) $-\frac{7}{20}$

27. What least value must be assigned to * so that the number $197 * 5462$ is invisible by 9?

- (A) 1 (B) 2
(C) 4 (D) None of these

28. In the number $357 * 25 *$ is divisible by both 3 and 5, then the missing digit in the unit's place and the thousandth place respectively are :

- (A) 0,6 (B) 5,6
(C) 5,4 (D) None of these

29. The sum of three consecutive odd number is always divisible by :

- I. 2 II. 3 III. 5 IV. 6
 (A) only I (B) only II
 (C) only I and III (D) only II and IV

30. $4^{61} + 4^{62} + 5^{63} + 5^{64}$ is divisible by :

- (A) 3 (B) 10
 (C) 11 (D) 13

31. Which of the following has fractions in ascending order ?

- (A) $\frac{2}{3}, \frac{3}{5}, \frac{7}{9}, \frac{9}{11}, \frac{8}{9}$ (B) $\frac{3}{5}, \frac{2}{3}, \frac{9}{11}, \frac{7}{9}, \frac{8}{9}$
 (C) $\frac{3}{5}, \frac{2}{3}, \frac{7}{9}, \frac{9}{11}, \frac{8}{9}$ (D) $\frac{8}{9}, \frac{9}{11}, \frac{7}{9}, \frac{2}{3}, \frac{3}{5}$

32. Which of the following fractions is less than $\frac{7}{8}$ and greater than $\frac{1}{3}$?

- (A) $\frac{1}{4}$ (B) $\frac{23}{24}$
 (C) $\frac{11}{12}$ (D) $\frac{17}{24}$

33. Simplify : $18 - [5 - \{6 + 2(7 - \frac{8-5}{})\}]$

- (A) 13 (B) 15
 (C) 27 (D) 32

34. $5 - \left[\frac{3}{4} + \left\{ 2 \frac{1}{2} - \left(0.5 + \frac{1}{6} - \frac{1}{7} \right) \right\} \right]$ is equal to :

- (A) $2 \frac{23}{84}$ (B) $3 \frac{1}{6}$
 (C) $3 \frac{3}{10}$ (D) $5 \frac{1}{10}$

35. If $2805 \div 2.55 = 1100$, then $280.5 \div 25.5 =$

- (A) 1.1 (B) 1.01
 (C) 0.11 (D) 11

36. The least number by which 294 must be multiplied to make it a perfect square, is :

- (A) 2 (B) 3
 (C) 6 (D) 24

37. The number of prime factors of $(3 \times 5)^{12} (2 \times 7)^{10} (10)^{25}$ is :

- (A) 47 (B) 60
 (C) 72 (D) None of these

38. On dividing a number 999, the quotient is 366 and th remainder is 103. The number is :

- (A) 364724 (B) 365387
 (C) 365737 (D) 366757

39. A number when divided by 3 leaves a remainder 1. When the quotient is divided by 2, it leaves a remainder 1. What will be the remainder when the number is divided by 6?

- (A) 6 (B) 3
 (C) 4 (D) 5

40. Evaluate : $\frac{8 - [5 - (-3 + 2)] \div 2}{|5 - 3| - |5 - 8| \div 3}$

- (A) 2 (B) 3
 (C) 4 (D) 5

41. The value of $\sqrt{214 + \sqrt{130 - \sqrt{88 - \sqrt{44 + \sqrt{25}}}}}$

[NTSE Stage - I/Raj./2007]

- (A) 14 (B) 15
 (C) 16 (D) 17

42. Teacher dictated the same dividend to four students. He gave them four different divisors 42, 56, 112 and 126 respectively, and asked to divided that dividend. Each student got the same remainder as 11, then the dividend is :

[NTSE Stage - I/Raj./2007]

- (A) LCM of all divisors + 11
 (B) HCF of all divisors + 11
 (C) Sum of all divisors + 11
 (D) None of these above

43. The value of $\sqrt{1 \frac{1}{2} [1 \frac{1}{2} - 1 \frac{1}{2} + (1 \frac{1}{2} - 1 \frac{1}{2} - 1 \frac{1}{4})]}$ is :

[NTSE Stage - I/Raj./2008]

- (A) $\frac{1}{2}$ (B) $\frac{1}{4}$
 (C) $\frac{1}{16}$ (D) $1 \frac{1}{5}$

44. If $\sqrt{121} = 1.1$ then $\sqrt{.000121}$ is equal to :

[NTSE Stage - I/Raj./2008]

- (A) 0.0011 (B) 0.011
 (C) 0.11 (D) 11.0

45. The value of $0.\bar{4}$ is :

[NTSE Stage - I/Raj./2008]

- (A) $\frac{4}{10}$ (B) $\frac{4}{9}$
 (C) $\frac{4}{100}$ (D) $\frac{9}{4}$

46. If a and b are natural numbers such that $\left(\frac{1}{a}\right)^{\frac{1}{b}} = 0.\bar{3}$

then the value of ab is :

[NTSE Stage - II/2007]

- (A) 81 (B) 24
 (C) 192 (D) 375

47. If the sum of the digits of a number $(10^n - 1)$ is 4707, when n is a natural number, then the value of n is :

[NTSE Stage - II/2007]

- (A) 477 (B) 523
 (C) 532 (D) 704

48. The last digit in the decimal representation of $\left(\frac{1}{5}\right)^{2000}$

is :

[NTSE Stage - II/2007]

- (A) 2 (B) 4
 (C) 5 (D) 6

49. Unit's digit in the product $(2137)^{753}$ is :

[NTSE Stage - II/2007]

- (A) 1 (B) 3
 (C) 7 (D) 9

50. The smallest possible integer x , for which $1260x = N^3$, where N is a positive integer is :

[NTSE Stage - II/2008]

- (A) 1470 (B) 2450
(C) 3675 (D) 7350

51. The number $\frac{1}{1+\sqrt{5}}$ lies between the numbers :

[NTSE Stage - II/2008]

- (A) $\frac{1}{3}$ and $\frac{1}{2}$ (B) $\frac{1}{2}$ and $\frac{1}{\sqrt{2}}$
(C) $\frac{1}{4}$ and $\frac{1}{3}$ (D) $\frac{1}{5}$ and $\frac{1}{4}$

52. If $x < -2$, then $|1 + |1 + x||$ equals :

[NTSE Stage - II/2008]

- (A) $2 + x$ (B) x
(C) $-x$ (D) $-(2 + x)$

53. A number when divided by 195 leaves a remainder 47. If the same number is divided by 15, then the remainder will be :

[NTSE Stage - II/2008]

- (A) 1 (B) 2
(C) 3 (D) 4



SURDS & EXPONENTS

Surds & Radical

Let a be a rational number and n be a positive integer such that $\sqrt[n]{a}$ is irrational, then $\sqrt[n]{a}$ is called as surd of the order n and a is called the radical.

$\sqrt[n]{a}$ is a surd if a is a rational number and $\sqrt[n]{a}$ is an irrational number.

Laws of radicals :

$$(i) \left(\sqrt[n]{a}\right)^n = \left(a^{\frac{1}{n}}\right)^n = a^{\frac{1}{n} \times n} = a.$$

$$(ii) \sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

For example : $\sqrt[3]{2} \cdot \sqrt[3]{4} = \sqrt[3]{2 \times 4} = \sqrt[3]{2^3} = (2^3)^{\frac{1}{3}} = 2$

$$(iii) \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

$$(iv) \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m} = a^{m/n}$$

$$(v) \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$$

Rationalising Factor :

If the product of two surds is a rational number, then each surd is called a rationalising factor (RF) of the other.

Rationalisation of surds :

The process of converting a surd into rational number by multiplying it with a suitable RF, is called the rationalisation of the surd.

Monomial surds and their RF:

The general form of a monomial surd is $K \sqrt[n]{a}$ and its RF is $a^{\frac{1}{n}}$.

Binomial surds and their RF:

The surds of the types : $a + \sqrt{b}, a - \sqrt{b}, \sqrt{a} + \sqrt{b}$ and $\sqrt{a} - \sqrt{b}$ are called binomial surds.

Conjugate Surds :

The binomial surds which differ only in sign between the terms separating them are known as conjugate surds. In binomial surds, the conjugate surds are RF of each other.

For example :

$$(i) \text{ RF of } \sqrt{a} + \sqrt{b} \text{ is } \sqrt{a} - \sqrt{b}$$

$$(ii) \text{ RF of } \sqrt{a} - \sqrt{b} \text{ is } \sqrt{a} + \sqrt{b}$$

Trinomial surds and their RF:

A surd which consists of three terms, at least two of which are monomial surds, is called a trinomial surd.

Exponents

The repeated multiplication of the same factor can be written in a more compact form called exponential form.

Laws of exponents :

If a is any non-zero rational number and m, n are whole numbers, then

(i) On the same base in multiplication, powers are added.

$$a^m \times a^n = a^{m+n}$$

For example : $3^2 \times 3^4 = 3^{2+4} = 3^6$.

(ii) On the same base in division, powers are subtracted

$$\frac{a^m}{a^n} = a^{m-n}$$

For Example : $\frac{3^5}{3^2} = 3^{5-2} = 3^3$

$$(iii) \frac{a^m}{a^n} = \frac{1}{a^{n-m}}, n > m$$

For Example : $\frac{2^3}{2^4} = \frac{1}{2^{4-3}} = \frac{1}{2}$

$$(iv) (a^m)^n = a^{mn}$$

For Example : $(2^2)^3 = 2^{2 \times 3} = 2^6$.

$$(v) a^n \times a^{-n} = a^0 = 1$$

$$(vi) a^m \times b^m = (ab)^m$$

For example : $2^2 \times 3^2 = (2 \times 3)^2 = 6^2 = 36$.

EXERCISE

1. The value of $\frac{2^{m+3} \times 3^{2m-n} \times 5^{m+n+3} \times 6^{n+1}}{6^{m+1} \times 10^{n+3} \times 15^m}$ is equal to :
 (A) 0 (B) 1
 (C) 2^m (D) None of these
2. The value of $\left[\left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^3\right] \times 2^6$ is :
 (A) 1 (B) 2
 (C) 3 (D) 4
3. If $3^x = 500$, then the value of 3^{x-2} is :
 (A) $\frac{100}{9}$ (B) $\frac{100}{9}$
 (C) $\frac{500}{9}$ (D) $\frac{500}{3}$
4. If $\frac{p}{q} = \left(\frac{2}{3}\right)^3 \div \left(\frac{3}{2}\right)^{-3}$ then the value of $\left(\frac{p}{q}\right)^{-10}$ is :
 (A) 1 (B) 0
 (C) cannot be determined (D) None of these
5. $\sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}$ is :
 (A) equal 1
 (B) lies between 0 and 1
 (C) lies between 1 and 2 (D) is greater than 2

6. The value of $\sqrt[3]{343} \times \sqrt{-64}$ is :
 (A) 28 (B) -28
 (C) 18 (D) -18
7. The value of $(3^0 - 4^0) \times 5^2$ is :
 (A) 25 (B) 0
 (C) -25 (D) None of these
8. The solution of $3^{3x-5} = -\frac{1}{9^x}$ is :
 (A) $\frac{5}{2}$ (B) 5
 (C) 1 (D) $\frac{7}{2}$
9. The value of $\left[(-2)^{(-2)}\right]^{(-3)}$ is :
 (A) 64 (B) 32
 (C) cannot be determined (D) None of these
10. Simplify : $(32)^{\frac{-2}{5}} \div (125)^{\frac{-2}{3}}$
 (A) $\frac{4}{25}$ (B) $\frac{25}{4}$
 (C) $\frac{2}{5}$ (D) $\frac{5}{2}$
11. The value of $\frac{4\sqrt{3}-4}{\sqrt{3}+1} - \frac{2-\sqrt{3}}{2+\sqrt{3}}$ is :
 (A) 0 (B) 1
 (C) $8\sqrt{3}$ (D) 15
12. $(64)^{\frac{-2}{3}} \times \left(\frac{1}{4}\right)^{-3}$ equals to :
 (A) 4 (B) $\frac{1}{4}$
 (C) 1 (D) 16
13. The value of $\frac{(5)^{0.25} \times (125)^{0.25}}{(256)^{0.10} \times (256)^{0.15}}$ is :
 (A) $\frac{\sqrt{5}}{2}$ (B) $\frac{5}{4}$
 (C) $\frac{25}{2}$ (D) $\frac{25}{16}$
14. If $x + \frac{1}{x} = 2$ then $x^8 + \frac{1}{x^8}$ is equal to :
 (A) 1 (B) 3
 (C) 2 (D) $\sqrt{2}$
15. The value of $\frac{(0.6)^0 - (0.1)^{-1}}{\left(\frac{3}{2^3}\right)^{-1} \left(\frac{3}{2}\right)^3 + \left(\frac{-1}{3}\right)^{-1}}$ is :
 (A) $\frac{3}{2}$ (B) $-\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $-\frac{1}{2}$
16. The value of $\frac{\sqrt[6]{0.001} \sqrt{x^{216}}}{\sqrt{10}}$ is :
 (A) $(10x)^{36}$ (B) $\frac{x^6}{10^3}$
 (C) $\frac{x^{36}}{10}$ (D) None of these
17. If $a = \frac{1}{3-2\sqrt{2}}, b = \frac{1}{3+2\sqrt{2}}$ then the value of $a^2 + b^2$ is:
 (A) 34 (B) 35
 (C) 36 (D) 37
18. If $a = \frac{1}{3-2\sqrt{2}}, b = \frac{1}{3+2\sqrt{2}}$ then the value of $a^3 + b^3$ is
 (A) 193 (B) 194
 (C) 198 (D) 196
19. If $x = (7+4\sqrt{4})$ then the value of $\sqrt{x} + \frac{1}{\sqrt{x}}$ is :
 (A) 8 (B) 6
 (C) 5 (D) 4
20. The simplest rationalising factor of $\sqrt[4]{48}$ is :
 (A) $\sqrt[4]{9}$ (B) $\sqrt[4]{27}$
 (C) $\sqrt[3]{9}$ (D) None of these
21. The exponential form of $\sqrt{\sqrt{2} \times \sqrt{3}}$ is :
 (A) $6^{-1/2}$ (B) $6^{1/2}$ (C) $6^{1/4}$ (D) 6
22. The value of $\frac{6^n \times 2^{2n} \times 3^{3n}}{30^n \times 3^{2n} \times 3^{3n}}$ is equal to :
 (A) 1 (B) $(0.3)^n$ (C) 3^{-n} (D) $(0.3)^n$
23. The value of $\frac{2^{1/2} \times 3^{1/3} \times 4^{1/4}}{10^{-1/5} \times 5^{3/5}} \div \frac{4^{-2/3} \times 5^{-7/5}}{4^{-3/5} \times 6^{-1/3}}$ is :
 (A) 10 (B) 1 (C) 6 (D) 18
24. The value of $\left[\left(\frac{-1}{3}\right)^{-2}\right]^{-2} \times \left[\left(\frac{2}{3}\right)^2\right]^{-2} \div \left[\left(\frac{3}{2}\right)^{-1}\right]^{-2}$ is
 [NTSE Stage-I/Raj. /2007]
 (A) 81 (B) 36 (C) $\frac{1}{81}$ (D) $\frac{1}{36}$
25. If $(4)^3 \times (6)^4 \times (10)^5 = 2^x \times 3^y \times 5^z$, then the value of $x + y + z$ is :
 [NTSE Stage - I/raj./2008]
 (A) 12 (B) 15 (C) 20 (D) 24
26. Which of the following is equal to 1 ?
 [NTSE Stage - II/raj./2007]
 (A) $\frac{(0.11)}{(1.1)^2 \times 0.1}$ (B) $\frac{(1.1)^2}{(11)^2 \times 0.1}$
 (C) $\frac{(0.011)^2}{(1.1)^2 \times (0.01)^2}$ (D) $\frac{(0.11)^2}{(11)^2 \times 0.01}$
27. If $\left(\frac{1}{17}\right)^0 - (64)^{\frac{1}{2}} - (-32)^{\frac{4}{5}} = \frac{b}{a}$, where a and b are positive integers, then the smallest value of $a - b$ is :
 [NTSE Stage - II/2007]
 (A) 1 (B) 2 (C) 3 (D) 4

EXERCISE

1. Find the CF of 108, 288 and 360.
(A) 18 (B) 24
(C) 36 (D) 72
2. Find the H.C.F. of 513, 1134 and 1215.
(A) 108 (B) 54
(C) 27 (D) 168
3. What is the HCF of $\frac{1}{5}$, $\frac{2}{7}$ and $\frac{3}{11}$?
(A) $\frac{1}{385}$ (B) 6
(C) $\frac{1}{35}$ (D) $\frac{5}{77}$
4. Find the H.C.F. of 3.9, 6.6, 8.22.
(A) 0.03 (B) 0.3
(C) 0.6 (D) 1.2
5. Find the L.C.M. of 72, 108 and 2100.
(A) 36866 (B) 38680
(C) 36800 (D) 37800
6. Find the L.C.M. of 16, 24, 36 and 54.
(A) 432 (B) 342
(C) 452 (D) 436
7. Find the L.C.M. of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{1}{27}$.
(A) $\frac{4}{9}$ (B) $\frac{30}{7}$
(C) $\frac{80}{3}$ (D) $\frac{80}{9}$
8. The smallest fraction, which each of $\frac{6}{7}$, $\frac{5}{14}$, $\frac{10}{21}$ will divide exactly, is :
(A) $\frac{30}{7}$ (B) $\frac{30}{98}$
(C) $\frac{60}{147}$ (D) $\frac{50}{294}$
9. Reduce $\frac{391}{667}$ to lowest terms.
(A) $\frac{17}{29}$ (B) $\frac{18}{29}$
(C) $\frac{19}{29}$ (D) None of these
10. The LCM of two numbers is 864 and their HCF is 144. If one of the numbers is 288, the other numbers is :
(A) 576 (B) 1296
(C) 432 (D) 144
11. LCM of two distinct natural numbers is 211. What is their HCF ?
(A) 37 (B) 211
(C) 1 (D) Data insufficient
12. The L.C.M. of two number is 12 times H.C.F. The sum of The H.C.F. and L.C.M. is 403. If one of the numbers is 93, then the other number is :
(A) 134 (B) 124
(C) 128 (D) 310
13. The LCM of two numbers is 567 and their HCF is 9. If the difference between the two numbers is 18, find the two numbers :
(A) 36 and 18 (B) 78 and 60
(C) 63 and 81 (D) 52 and 34
14. Two numbers are in the ratio of 15 : 11. If their H.C.F. is 13, find the numbers.
(A) 195, 143 (B) 195, 123
(C) 175, 123 (D) 163, 115
15. Find a number greater than 3 which when divided by 4, 5 and 6 always leaves the same remainder 3.
(A) 60 (B) 63
(C) 57 (D) 123
16. In a school 437 boys and 342 girls have been divided into classes, so that each class has the same number of students and no class has boys and girls mixed. What is the least number of classes needed ?
(A) 41 (B) 18
(C) 23 (D) None of these
17. Find the least number which when divided by 6, 7, 8, 9 and 10 leaves remainder 1.
(A) 2531 (B) 2521
(C) 5041 (D) 7561
18. How many three-digit numbers would you find, which when divided by 3, 4, 5, 6, 7 leave the remainders 1, 2, 3, 4, and 5 respectively ?
(A) 4 (B) 2
(C) 1 (D) 2
19. Six strings of violin start vibrating simultaneously and they vibrate at 3, 4, 5, 6, 10 and 12 times in a minute, find :
i. After how much time will all six of them vibrate together ?
ii. How many times will they vibrate together in 30 min ?
(A) 60 min, 31 times (B) 60 s, 31 times
(C) 120 s, 15 times (D) None of these
20. The traffic lights of three different road crossings change after every 48 sec, 72 sec. and 108 sec. respectively. If all change simultaneously ?
(A) 8 : 17 : 10 (B) 8 : 27 : 12
(C) 8 : 27 : 02 (D) None of these

21. H.C.F. of 3240, 3600 and a third number is 36 and their L.C.M. is $2^4 \times 3^5 \times 5^2 \times 7^2$. Then the third number is :
- (A) $2^2 \times 3^5 \times 7^2$ (B) $2^2 \times 5^3 \times 7^2$
(C) $2^5 \times 5^2 \times 7^2$ (D) $2^3 \times 3^5 \times 7^2$
22. The sum of two numbers is 216 and their H.C.F. is 27. The numbers are :
- (A) 27, 189 (B) 81, 189
(C) 108, 108 (D) 154, 162
23. Among a number of students, 1001 pens and 910 pencils is distributed in such a way that each student gets the equal number of pens and equal number of pencils. Find the maximum number of students.
- (A) 91 (B) 910
(C) 1001 (D) 1911
24. A rectangular courtyard 3.78 metres long and 5.25 metres wide is to be paved exactly with square tiles, all of the same size. What is the largest size of the tile which could be used for the purpose ?
- (A) 14 cm (B) 21 cm
(C) 42 cm (D) None of these
25. A, B and C start at the same time in the same direction to run around a circular stadium. A complete a round in 252 seconds, B in 308 seconds and C in 198 seconds, all starting at the same point. After what time will they meet again at the starting point ?
- (A) 26 minutes 18 seconds
(B) 42 minutes 36 seconds
(C) 45 minutes
(D) 46 minutes 12 seconds



ALGEBRA

Introduction

Algebra is that branch of Mathematics in which letter represent any value which we can assign according to our requirement. These letters are generally of two types : constants and variables (or literal numbers).

Polynomial

- ❖ An algebraic expression containing only one variable with the powers of this variable as non negative integers i.e. whole numbers, is called a polynomial in that variable.
- ❖ An algebraic expression (x) of the form $P(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ where $a_0, a_1, a_2, \dots, a_n$ are real numbers and n is any non negative integer in called a polynomial is x.
- ❖ Algebraic expression P(x) is called polynomial of degree n in variable x.

Polynomial One Variable

- ❖ $a_0 + a_1x$ is called a polynomial in one variable expression is x.
- ❖ Degree of Polynomial in one variable : The degree of a polynomial in one variable is the greatest power (or index or exponent) of the variable.
- ❖ Constant Polynomial : If the polynomial is a_0 ($a_0 \neq 0$), then it is called a constant polynomial and its degree is 0 (Zero).

Ex.1 Find the degree of following polynomial

(i) $-9, \frac{15}{4}, -\frac{13}{3}$

(ii) $-45x, 2y + \frac{9}{4}$

Sol. (i) All polynomials of degree 0.
(ii) All polynomials of degree 1 in x and y respectively.

- ❖ Linear Polynomials : A polynomial of degree 1 in one variable is called linear polynomial. It is in the form of $f(x) = ax + b$

For zero of this polynomial, as $+ b = 0$ is $x = \frac{-b}{a}$

So, zero of polynomial is $\frac{-b}{a}$

\Rightarrow It will intersect X-axis at $(\frac{-b}{a}, a)$

Ex.2 Find the zero of linear polynomial $p(x) = x + 2$

Sol. $p(x) = x + 2$

For zero of the polynomial $p(x) = x + 2$ is

$$p(x) = x + 2 = 0$$

$$\Rightarrow x = -2$$

zero of polynomial p(x) is -2.

- ❖ **Quadratic Polynomial** : A polynomial of degree two in one variable is quadratic polynomial. It is in the form of $f(x) = ax^2 + bx + c = 0, a \neq 0$
Quadratic polynomial can be factorize in two linear factors.
- ❖ **Cubic polynomial** : A polynomial of degree three is called a cubic polynomial.

Polynomial in Two or More Variables

- ❖ An algebraic expression containing two or more variables with the powers of the variables as non negative integers (or nature numbers), is called as polynomial in two or more variables.
- ❖ **Degrees of a Polynomial in Two or More Variable** : Take the sum of the powers (or indices or exponents) of the variables in each term; the greatest sum is the degree of the polynomial. The sum of the powers (or indices or exponents) of the variables in each term is called the degree of that term.

H.C.F. and L.C.M. of polynomials

- ❖ A polynomial D(x) is a divisor of the polynomial P(x) if it is a factor of P(x). Where Q(x) is another polynomial such that $P(x) = D(x) \times Q(x)$
- ❖ HCF/GCD (Greatest Common Divisor) : A polynomial h(x) is called the HCF or GCD of two or more given polynomials, if h(x) is a polynomial of highest degree dividing each of one of the given polynomials.
- ❖ L.C.M. (Least Common Multiple) : A polynomial P(x) is called the LCM of two or more given polynomials, if is a polynomial of smallest degree which is divided by each one of the given polynomials. For any two polynomials P(x) and Q(x). we have :
 $P(x) \times Q(x) = [\text{HCF of } P(x) \text{ and } Q(x)] \times [\text{LCM of } P(x) \text{ and } Q(x)]$

Factors

- ❖ When an algebraic expression can be written as the product of two or more expression then each of these expression is called a factor.
- ❖ Factorization : The process of writing a given algebraic expression at the product of two or more factors is called factorization.

Important Formulae

- ❖ $(a + b)^2 = a^2 + 2ab + b^2$
- ❖ $(a - b)^2 = a^2 - 2ab + b^2$
- ❖ $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a + b)$
- ❖ $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a - b)$
- ❖ $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- ❖ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- ❖ $a^2 - b^2 = (a + b)(a - b)$
- ❖ $a^3 + b^3 + c^3 - 3abc$
 $= (a + b + c)(a^2 + b^2 + c^2 - b - bc - ca)$
- ❖ $a^3 + b^3 + c^3 = 3abc$ if $a + b + c = 0$
- ❖ $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Remainder Theorem

If a polynomial $P(x)$ is divided by $(x - a)$, then remainder is $P(a)$.

Ex.3 If $P(x) = x^3 + x^2 + 2x + 3$ is divided by $x + 2$, then find its remainder.

Sol. Remainder is $P(-2)$

$$\begin{aligned} P(x) &= x^3 + x^2 + 2x + 3 \\ &= -8 + 4 - 4 + 3 = -12 + 7 = -5 \\ \text{Remainder} &= -5 \end{aligned}$$

Factor Theorem

- ❖ If $(x + a)$ is a factor of polynomial $P(x)$, then remainder = 0
 $\Rightarrow P(-a) = 0$

Ex.4 Show that $(x - 3)$ is a factor of the polynomial $x^3 - 3x^2 + 4x - 12$.

Sol. If $(x - 3)$ is a factor of polynomial $P(x) = x^3 - 3x^2 + 4x - 12$ then remainder = 0

$$\begin{aligned} \therefore P(3) &= 0 \\ P(3) &= 3^3 - 3 \times 3^2 + 4 \times 3 - 12 \\ &= 27 - 27 + 12 - 12 = 0 \end{aligned}$$

As remainder $P() = 0$.

$\therefore (x - 3)$ is a factor of polynomial $P(x)$

Linear Equation One variable

An equation involving a variable in first degree is called a linear equation in one variable. It is of the form $ax + b = 0$, $a \neq 0$, x is variable

$$x = -\frac{b}{a} \text{ is the root or solution of this equation.}$$

Rules for solving a linear equation :

Rule 1 : Same quantity (number) can be added to both sides on an equation without changing the equality.

Rule 2 : Same quantity can be subtracted from both sides of an equation without changing the equality.

Rule : 3 Both sides of an equation may be multiplied by the same non-zero number without changing the equality.

Rule 4 : Both sides of an equation may be divided by the same non-zero number without changing the equality.

Rule 5 : (Transposition) Any term of an equation may be taken to the other side with the sign changed. This process is called transposition.

Ex.5 Solve $7x - 3 = 2x + 7$

Sol. $7x - 2x = 3 + 7$

$$\Rightarrow 5x = 10$$

$$\Rightarrow x = 2$$

Linear Equation in Two Variable

- ❖ Linear equation in two variable is of the form $ax + by + c = 0$, $a \neq 0$, $b \neq 0$.

- ❖ To solve the linear equation in two variables, we require two equations.

Ex.6 Solve : $3x - 2y = 4$, $x + y - 3 = 0$

Sol. $3x - 2y = 4$ (i)

$$x + y - 3 = 0 \text{(ii)}$$

Put this value of x in (i)

$$3x - 2y = 4$$

$$\Rightarrow 3(3 - y) - 2y = 4$$

$$\Rightarrow 9 - 3y - 2y = 4$$

$$\Rightarrow -5y = -5$$

$$\Rightarrow y = 1$$

Put $y = 1$ in eq. (ii).

$$\Rightarrow x + 1 - 3 = 0$$

$$\Rightarrow x - 2 = 0$$

$$\Rightarrow x = 2$$

\therefore Solution of this system of equations is $x = 2$, $y = 1$. These two linear equations represent two lines intersecting at the point $(2, 1)$

Quadratic Equation

- ❖ An equation of the form $ax^2 + bx + c = 0$, where a, b, c are constant and $a \neq 0$, is called a quadratic equation in one variable 'x'.

- ❖ Thus a quadratic equation is a polynomial equation in one variable x and is of degree 2.

- ❖ In a quadratic equation $ax^2 + bx + c = 0$, ax^2 is called second degree term, bx is called First degree term, c is called constant term or absolute term. a is called coefficient of x^2 , b is called coefficient of x .

- ❖ A number α is a root of the quadratic equation $ax^2 + bx + c = 0$ if it satisfies the equation.

i.e. if we replace x by α , both the sides of the equation become equal i.e. $a\alpha^2 + b\alpha + c = 0$

Ex.7 Show that 2 is a root of the equation $5x^2 - 7x - 6 = 0$.

Sol. $\therefore 5(2)^2 - 7(2) - 6 = 0$ i.e. $20 - 14 - 6 = 0$

$$\text{i.e. } 0 = 0 \text{ which is true.}$$

Ex.8 Show that 3 is not a root of the equation $5x^2 - 7x - 6 = 0$.

Sol. $\therefore 5(3)^2 - 7(3) - 6 = 0$

$$\text{i.e. } 45 - 21 - 6 = 0$$

i.e. $18 \neq 0$ So, 3 is not a root of the given equation.

Solution of a Quadratic Equation.

- ❖ Roots of a quadratic equation is called solution of the quadratic equation.
- ❖ A quadratic equation has at the most two real roots. Therefore a quadratic equation has either no real root or two equal real roots or two distinct real roots.
- ❖ The set of roots of the quadratic equation is called solution set of the quadratic equation.

Method to solve a quadratic equation by using Factors

The following steps are generally used:

(i) Write the quadratic equation as $ax^2 + bx + c = 0$ i.e. making right hand side zero and clearing all fractions on left hand side.

(ii) Use the concept that $ab = 0$ implies either $a = 0$ or $b = 0$ (or both $a = 0$ and $b = 0$).

Therefore put each linear factor equal to zero and solve the resulting leaner equations.

By Using Quadratic Formula :

Solve the quadratic equation in general from $ax^2 + bx + c = 0$.

$$\text{We have, } ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- ❖ By comparison with general quadratic we get the required solution.
- ❖ Find the discriminate of the quadratic equation.
 $D = b^2 - 4ac$
- ❖ Now find the roots of the equation by given equation

$$x = \frac{-b + \sqrt{D}}{2a}, \frac{-b - \sqrt{D}}{2a}$$

Ex.9 $x^2 - 36 = 0$

Given $x^2 - 36 = 0$

$\Rightarrow x^2 - 6^2 = 0$

$\Rightarrow (x - 6)(x + 6) = 0$

$\Rightarrow x - 6 = 0$ or $x + 6 = 0$

$\Rightarrow x = 6$ or $x = -6$

Thus the roots are -6 and 6.

Ex.10 $x^2 + 5x + 6 = 0$

$\Rightarrow x^2 + 3x + 2x + 6 = 0$

$\Rightarrow x(x + 3) + 2(x + 3) = 0$

$\Rightarrow (x + 3)(x + 2) = 0$

$\Rightarrow x + 3 = 0$ or $x + 2 = 0$

$\Rightarrow x = -3$ or $x = -2$

Thus, the roots are -3, and -2.

Ex.11 Solve the quadratic equation $x^2 - 7x - 5 = 0$.

Sol. Comparing the given equation with $ax^2 - bx + c = 0$, we find that $a = 1$, $b = -7$ and $c = -5$.

Therefore, $D = (-7)^2 - 4 \times 1 \times (-5) = 49 + 20 = 69$

Since D is positive, the equation has two roots given

by $\frac{7 + \sqrt{69}}{2}, \frac{7 - \sqrt{69}}{2}$

$\Rightarrow x = \frac{7 + \sqrt{69}}{2}, \frac{7 - \sqrt{69}}{2}$ are the required solutions.

EXERCISE

1. The factors of $\frac{x^2}{4} - \frac{y^2}{9}$ are :
(A) $\left(\frac{x}{y} + \frac{y}{9}\right), \left(\frac{x}{4} - \frac{y}{9}\right)$ (B) $\left(\frac{x}{2} + \frac{y}{9}\right), \left(\frac{x}{2} - \frac{y}{9}\right)$
(C) $\left(\frac{x}{2} + \frac{y}{3}\right), \left(\frac{x}{2} - \frac{y}{3}\right)$ (D) None of these
2. The value of $\frac{0.76 \times 0.76 \times 0.76 + 0.24 \times 0.24 \times 0.24}{0.76 \times 0.76 - 0.76 \times 0.24 + 0.24 \times 0.24}$ is :
(A) 0.52 (B) 1
(C) 0.01 (D) 0.1
3. The solution of $\frac{2x+1}{3x-1} = \frac{3}{2}$ is :
(A) $x = 1$ (B) $x = -1$
(C) $x = 2$ (D) $x = -3$
4. In an examination a student was asked to find $\frac{3}{14}$ th of a certain number. By mistake, he found $\frac{3}{4}$ of it. His answer was 150 more than correct answer. The number given to him is :
(A) 290 (B) 280
(C) 240 (D) 180
5. A man is 5 years older than his wife and the wife is now thrice as old as their daughter, who is 10 years old. How old was the man when his daughter was born ?
(A) 20 years (B) 23 years
(C) 25 years (D) 30 years
6. If $x + y = 3$ and $xy = 2$, then the value of $x^3 - y^3$ is equal to
(A) 6 (B) 7
(C) 8 (D) 0
7. A farmer divides his herd of n cows among his four sons so that first son gets one half the herd, the second son gets one fourth, the third son gets one fifth, and the fourth son gets 7 cows, then n is :
(A) 180 (B) 140
(C) 240 (D) 100
8. If the difference of the squares of two numbers is 45, the square of the smaller number is 4 times the larger number, then the number are
(A) 9, 6, or 9, -6 (B) 5, 6 or 5, -6
(C) 9, 5 or 9, -5 (D) None of these
9. If $x = \frac{1}{2}$, then the value of $x + \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}}$ is :
(A) $\frac{5}{4}$ (B) $\frac{4}{5}$
(C) $\frac{3}{4}$ (D) None of these

10. If $2^{2x-y} = 32$ and $2^{x+y} = 16$ then $x^2 + y^2$ is equal to :
 (A) 9 (B) 10
 (C) 11 (D) 13
11. Factors of $2(a+b)^2 - 9(a+b) - 5$ are :
 (A) $a + b + 5, 2a + 2b - 1$
 (B) $a + b - 5, 2a + 2b + 1$
 (C) $a - b + 5, 2a - 2b + 5$
 (D) None of these
12. If $x - \frac{1}{x-2} = 2 - \frac{1}{x-2}$ then x is equal to :
 (A) 1 (B) 2
 (C) 3 (D) None of these
13. The value of $\frac{x^{a+b} \cdot x^{b+x} \cdot x^{c+a}}{(x^a \cdot x^b \cdot x^c)^2}$ is :
 (A) x^2 (B) x^{a+b+c}
 (C) x^{abc} (D) x^0
14. If two numbers differ by 3 and their product is 504, then the numbers are :
 (A) 21, 24, or -24, -21
 (B) 30, 31 or -30, -31
 (C) 40, 41 or -40, -41
 (D) None of these
15. Ramu's father is thrice old as Ramu. If father's age is 45 years then Ramu's age is :
 (A) 45 yrs (B) 30 yrs
 (C) 15 yrs (D) 10 yrs
16. In a piggy bank the number of 25 paise coins are five times the number of 50 paise coins. If there are 120 coins find the amount in the bank ?
 (A) Rs. 25 (B) Rs. 10
 (C) Rs. 35 (D) Rs. 40
17. If quotient = $3x^2 - 2x + 1$, remainder = $2x - 5$ and divisor = $x + 2$, then the dividend is :
 (A) $3x^3 - 4x^2 + x - 3$ (B) $3x^3 - 4x^2 - x + 3$
 (C) $3x^3 + 4x^2 - x + 3$ (D) $3x^3 + 4x^2 - x - 3$
18. What must be subtracted from $x^4 + 2x^2 - 3x + 7$ to get $x^3 + x^2 + x - 1$?
 (A) $x^4 - x^3 + x^2 - 4x + 8$
 (B) $x^3 + x^2 - x + 3$
 (C) $3x^3 + 4x^2 - x + 3$
 (D) $3x^3 + 4x^2 - x - 3$
19. If $(3x - 4)(5x + 7) = 15x^2 - ax - 28$, then $a =$ _____
 (A) 1 (B) -1
 (C) -2 (D) None
20. Solve for x : $\frac{x-a}{b+c} + \frac{x-b}{c+a} + \frac{x-c}{a+b} = 3$
 (A) $\frac{1}{2}(x+b+c)$ (B) $a+b+c$
 (C) $3(a+b+c)$ (D) $2(abc)$
21. If $\frac{4}{y} + \frac{3}{x} = \frac{1}{xy}$ and $\frac{5}{y} - \frac{7}{x} = \frac{12}{xy}$ find $x + y$
 (A) 2 (B) 1
 (C) 0 (D) -1
22. The value of $\left(\frac{a^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$ is equal to
 (A) 0 (B) 1
 (C) x (D) x^{a+b+c}
23. If $2^{x+3} \cdot 4^{2x-5} = 2^{3x+7}$, then the value of x is :
 (A) 3 (B) 4
 (C) 6 (D) 7
24. What are the roots of the equation $x^2 - x - 6 = 0$?
 (A) $\left(0, \frac{1}{2}\right)$ (B) (-2, 3)
 (C) $\left(\frac{1}{2}, 1\right)$ (D) $\left(2, \frac{1}{2}\right)$
25. Choose the correct option for the roots of $\frac{1}{x} - \frac{3}{4} + \frac{1}{2+x} = 0$
 (A) $\left(2, \frac{3}{4}\right)$ (B) (5, 4)
 (C) (-8, 0) (D) $\left(-\frac{4}{3}, 2\right)$
26. The roots of $\frac{x}{x-1} + \frac{x-1}{x} = 2\frac{1}{2}$ are
 (A) (5, 4) (B) (5, -4)
 (C) (1, 2) (D) (-1, 2)
27. $5^{x-1} + 5^x + 5^{x+1}$ then, the value of x, where is a positive integer, is :
 (A) 1 (B) 3
 (C) 2 (D) Can't be determined
28. Which of the following equations is not linear equation
 (A) $2x + 3 = 7x - 2$ (B) $\frac{2}{3}x + 5 = 3x - 4$
 (C) $x^2 + 3 = 5x - 3$ (D) $(x-2)^2 = x^2 + 8$
29. If $p = 3x + 1$, $q = \frac{1}{3}(9x + 13)$ and $p : q = 6 : 5$, then find x.
 (A) 7 (B) -7
 (C) 3 (D) None of these
30. If $(a^2 + b^2)^3 = (a^3 + b^3)^2$ then $\frac{a}{b} + \frac{b}{a} =$
 (A) $\frac{2}{3}$ (B) $\frac{3}{2}$
 (C) $\frac{5}{6}$ (D) $\frac{6}{5}$
31. The HCF of $(x^2 - 4)$, $(x^2 - 5x - 6)$ and $(x^2 + x - 6)$ is :
 (A) 1 (B) $x + 2$
 (C) $x - 2$ (D) $(x^2 + x - 6)$
32. The LCM of $(a^3 + b^3)$ and $(a^4 - b^4)$ is:
 (A) $(a^3 + b^3)(a^2 + b^2)(a - b)$
 (B) $(a^3 + b^3)(a^2 + b^2)(+b)$
 (C) $(a^3 + b^3)(a^2 + b^2 + ab)(a + b)$
 (D) $(a^3 + b^3)(a^2 - b^2)(a - b)$

33. If $\frac{3}{4}x = -7 + x$, then the value of x is :
(A) 4 (B) $-\frac{7}{3}$
(C) -28 (D) 28
(NTSE Stage-I/Raj./2007)
34. If $x = 1$, $y = -1$ and $z = -$, then the value of $\frac{x^2yz^2}{3}$ is :
(A) $\frac{1}{3}$ (B) $-\frac{1}{3}$
(C) 1 (D) -1
(NTSE Stage-I/Raj./2007)
35. HCF and LCM of two expression are $(x - 6)$ and $(x + 6)(x - 1)(x - 6)$ respectively. If one of the expression is $x^2 - 7x + 6$, then the other expression is :
(A) $(x - x)(x - 1)$ (B) $(x + 6)(x - 1)$
(C) $(x + 6)(x - 6)$ (D) $(x^2 - 7x + 6)(x - 1)$
(NTSE Stage-I/Raj./2007)
36. If the expression $x^2 + K + \frac{1}{x^2}$ is perfect square, then the value of K is :
(A) 2x (B) 2
(C) 1 (D) $\frac{1}{2x}$
(NTSE Stage-I/Raj./2007)
37. Factors of $x^2 + ax + b$ are $(x - 7)$ and $(x + 9)$ then the value of a and b is :
(A) $a = 2, b = -63$ (B) $a = -2, b = 63$
(C) $a = -2, b = -63$ (D) $a = 2, b = 63$
(NTSE Stage-I/Raj./2007)
38. Three numbers x, y and z are such that $x = y \neq z$ but $x + y + z = 0$. Value of $\frac{z^2 - x^2}{z^2 + y^2}$ is :
(A) $-\frac{3}{5}$ (B) $\frac{4}{5}$
(C) $\frac{3}{5}$ (D) $\frac{5}{3}$
(NTSE Stage-I/Raj./2007)
39. The denominator of a fraction is greater than numerator by 6. If 3 is added to numerator and 2 is subtracted from denominator, the fraction becomes $\frac{6}{7}$, then the equation so formed is :
(A) $\frac{x+4}{x+3} = \frac{6}{7}$ (B) $\frac{x+3}{x+4} = \frac{6}{7}$
(C) $\frac{x}{x+6} + \frac{3}{-2} = \frac{6}{7}$ (D) $\frac{x}{x+6} + \frac{-2}{3} = \frac{6}{7}$
(NTSE Stage-I/Raj./2007)
40. The value of x in $\frac{x+1}{2} + \left(x + \frac{x-1}{3}\right) = 2$ is :
(A) 1 (B) 2
(C) 3 (D) 0
(NTSE Stage-I/Raj./2007)
41. A number lying between 10 and 100 is even times the sum of its digits. If 9 is subtracted from it, the digits of the number are reversed. Then the number is :
(A) 63 (B) 54
(C) 21 (D) 42
(NTSE Stage-II/2007)
42. The expression $x^2 - y^2 - z^2 + 2yz + x + y - z$ has a factor
(A) $x - y + z + 1$ (B) $-x + y + z$
(C) $x + y - z + 1$ (D) $x - y - z + 1$
(NTSE Stage-II/2007)
43. If $x + y + z = 0$, then a factor of the expression $(x + y)^3 + (y + z)^3 + (z + x)^3$ is :
(A) $3(x + y)(y + z)(z + x)$ (B) $-3xyz$
(C) $(x + y - z)$ (D) $(x - y + z)$
(NTSE Stage-II/2007)
44. If $\sqrt{x + \frac{x}{y}} = x\sqrt{\frac{x}{4}}$, Where x and y are positive real numbers, then y is equal to :
(A) $x + 1$ (B) $x^2 - 1$
(C) $x - 1$ (D) $x^2 + 1$
(NTSE Stage-II/2007)
45. If $x < -2$, then $|1 - |1 + x||$ equal
(A) $2 + x$ (B) x
(C) $-x$ (D) $-(2 + x)$
(NTSE Stage-II/2007)
46. If $xy + yz + zx = 1$, then the expression $\frac{x+y}{1-xy} + \frac{y+z}{1-yz} + \frac{z+x}{1-zx}$ is equal to :
(A) $\frac{1}{x+y+z}$ (B) $\frac{1}{xyz}$
(C) $x + y + z$ (D) xyz
(NTSE Stage-II/2007)
47. Then sum of the present ages of a father and his son is 99 years. When the father was as old as his son is now, his age was four times age of the son at that time. The ratio of the present ages of the son and the father is :
(A) 3 : 7 (B) 3 : 8
(C) 4 : 9 (D) 4 : 7
(NTSE Stage-II/2007)

PERCENTAGE

Important facts

1. Concept of Percentage : By a certain percent, we mean many hundredths. thus, x percent means x hundredths, written as x%

❖ To express x% as a fraction : We have $x = \frac{x}{100}$.

$$\text{Thus, } 20\% = \frac{20}{100} = \frac{1}{5} \text{ etc.}$$

❖ To express $\frac{a}{b}$ as a percent : We have

$$\frac{a}{b} = \left(\frac{a}{b} \times 100 \right) \% \quad \text{i.e.}$$

$$\frac{1}{4} = \left(\frac{1}{4} \times 100 \right) \% = 25\%; 0.6 = \frac{3}{5} = \left(\frac{3}{5} \times 100 \right) \% = 60\%$$

II. Important Formulae :

❖ If the price of a commodity increase by r% then the reduction in consumption so as not to increase the expenditure is :

It the price of a commodity decreases by r% then the increase in consumption so as not to decrease the

$$\text{expenditure is : } \left[\frac{r}{(100+r)} \times 100 \right] \%$$

❖ If the price of a commodity increase by r% then the reduction in consumption so as not to increase the expenditure is :

It the price of a commodity decreases by r% then the increase in consumption so as not to decrease the

$$\text{expenditure is : } \left[\frac{r}{(100-r)} \times 100 \right] \%$$

❖ If A is r% more the, B then B is less than A by :

$$\left[\frac{r}{(100-r)} \times 100 \right] \%$$

❖ If A is r% less than B, then B is more than A by :

$$\left[\frac{r}{(100-r)} \times 100 \right] \%$$

III. Results on Population : Let the population of a town be P now and suppose it increase at the rate of r%, per annum then :

❖ Population after n years = $P \left(1 + \frac{r}{100} \right)^n$

❖ Population n years ago = $\frac{P}{\left(1 + \frac{r}{100} \right)^n}$

IV. Results of Depreciation : Let the present value of a machine be P. Suppose it depreciates at the rate of r% per annum. Then :

❖ Value of the machine after n years = $P \left(1 - \frac{r}{100} \right)^n$

❖ Value of the machine n years ago = $\frac{P}{\left(1 - \frac{r}{100} \right)^n}$

EXERCISE

1. What percentage of 656 is 410 ?
(A) 63.5% (B) 60.5%
(C) 64.5% (D) 62.5%
2. Two-fifth of one-third of three-seventh of a number is 15. What is 40 percent of that number ?
(A) 72 (B) 84
(C) 105 (D) 140
3. A number, when 35 is subtracted from it, reduces to its 80 percent, What is four-fifth of that number ?
(A) 70 (B) 90
(C) 120 (D) 140
4. If A is 20% more than B, by what percent is B less than A?
(A) 16.66% (B) 26.36%
(C) 9.09% (D) 27.27%
5. Passing percentage marks in an exam is 40. A obtained 72 out of 200. By what percent did he fail ?
(A) 8 (B) 5 (C) 4 (D) 16
6. In order to compute 0.15% of a number, it must be multiplied by :
(A) 0.0015 (B) 0.015
(C) 0.15 (D) 1.5
7. A man spends 75% of his income. If his income is increased by 20% and he increased his expenditure by 10%. By what % will saving increased ?
(A) 50% (B) 55%
(C) 25% (D) 60%
8. If 25% of a number is subtracted from a second number, the second number reduces to its five-sixth. What is the ratio of the first number to the second number ?
(A) 1 : 3 (B) 2 : 3
(C) 3 : 2 (D) Data inadequate
9. A student has to obtain 33% of the total marks to pass. He got 125 marks and failed by 40 marks. The maximum marks are :
(A) 300 (B) 500
(C) 800 (D) 1000
10. 10% of the voters did not cast their votes in an election between two candidates. 10% of the votes polled were found invalid. The successful candidate got 54% of the valid votes and won by majority of 1620 votes. The number of voters enrolled on the voter's list was :
(A) 25000 (B) 33000
(C) 35000 (D) 40000

11. A scored 30% marks and failed by 15 marks. B scored 40% marks and obtained 35 marks more than those required to pass. The pass percentage is :
 (A) 33% (B) 38%
 (C) 43% (D) 46%
12. The price of wheat falls by 16%. By what percentage a person can increase the consumption of wheat so that his overall budget do not change ?
 (A) 16% (B) 18%
 (C) 18.5% (D) 19%
13. Peter earned 40% more money than Albert. Albert earned 20% less than Michael. Peter earned more than Michael by :
 (A) 10% (B) 12%
 (C) 20% (D) 25%
14. Milk contains 5% water. What quantity of pure milk should be added to 10 litres of milk to reduce this to 2%
 (A) 5 liters (B) 7 litres
 (C) Cannot be determined (D) None of these
15. Sales tax on cloth is reduced by 10% but the scale increase by 10%. What is the effect on the revenue earned as sales tax ?
 (A) Remains the same (B) Increase by 5.5%
 (C) Decreases by 5.5% (D) Decrease by 1%
16. A and B's salaries are together equal to Rs. 2100. A spends 80% of his salary and B spends 70%. If their saving are now in the proportion of 4:3, what is A's salary ?
 (A) Rs. 900 (B) Rs. 980
 (C) Rs. 1200 (D) Rs. 1400
17. In an examination, A gets 10% marks less than B and B gets 10% marks less than C. If A get 810 marks, what marks does C get ?
 (A) 900 (B) 945
 (C) 973 (D) 1000
18. 30% of 28% of 480 is the same as :
 (A) 15% of 56% of 240 (B) 60% of 28% of 240
 (C) 60% of 56% of 240 (D) None of these
19. In the cost of the book worth Rs. 50 is increased by Rs. 25, the rate of increase is :
 (A) 25% (B) 20%
 (C) 50% (D) 10%
20. A man spends Rs. 3500 and save $12\frac{1}{2}\%$ of his income. His monthly income is :
 (A) Rs. 4000 (B) Rs. 3800
 (C) Rs. 4200 (D) Rs. 4500
21. In an examination 52% candidates failed in English 42% in Maths and 17% in both. The percentage of those passed in both the subjects is :
 (A) 23% (B) 40%
 (C) 53% (D) 33%
22. The price of an item is increased by 20% and then decreased by 20%, the final price as compared to original price is :
 (A) 4% less (B) 4% more
 (C) 20% less (D) 20% more
23. The numerator of a fraction is increased by 20% and its denominator be diminished by 10%. The value of new fraction is $\frac{16}{27}$, then the fraction is :
 (A) $\frac{4}{9}$ (B) $\frac{3}{2}$
 (C) $\frac{3}{8}$ (D) $\frac{9}{4}$
24. A sample of 5 liters of glycerin is found to be adulterated to the extent of 20%. Find how much glycerin should be added to bring down percentage of impurity to 5%.
 (A) 10 liters (B) 25 liters
 (C) 15 liters (D) 20 liters
25. The daily wages of a worker increased by 20% but the number of hours worked by him also dropped by 20%. If originally he was getting Rs 200 per week then his wages per week now is :
 (A) Rs. 160 (B) Rs. 192
 (C) Rs. 210 (D) Rs. 198
26. If A is 25% of B then what % is B of A ?
 (A) 40 (B) 400
 (C) 20 (D) 140
27. Let x and y be two numbers such that $x = 6y$. Then y is less than x by : **[NTSE Stage-I/Raj./2007]**
 (A) 50% (B) 60%
 (C) 75% (D) $83\frac{1}{3}\%$
28. The price of sugar rises by 50%. By what fraction a family reduces the consumption of sugar, so that its expenditure does not increase ? **[NTSE Stage-I/Raj./2007]**
 (A) $\frac{1}{4}$ (B) $\frac{1}{3}$
 (C) $\frac{1}{2}$ (D) $\frac{2}{3}$
29. Area of a rectangle is A. If its length is reduced by 10% and its breadth is increased by 10% then which of the following statements is true ? **[NTSE Stage-II/2007]**
 (A) A remains unchanged (B) A is decreased by 1%
 (C) A is decreased by 0.1% (D) A is increased by .1%
30. If from time $t = 0$, to time $t = 1$, a population in town increases by $i\%$ and from time $t = 1$, to time $t = 2$, the population increases by $j\%$ then the increase in population from time $t = 0$ to time $t = 2$, is : **[NTSE Stage-II/2007]**
 (A) $(i + j)\%$ (B) $(ij)\%$
 (C) $(i + ij)\%$ (D) $\left(i + j + \frac{ij}{100}\right)\%$

PROFIT AND LOSS

Profit and Loss

- ❖ **Cost Price** : The price at which an article is made is known as its cost price. The cost price is abbreviated as C.P.
- ❖ **Selling Price** : The price at which an article is sold is known as its selling price. The selling price is abbreviated as S.P.
- ❖ **Profit** : If the selling price (S.P.) of an article is greater than the cost price (C.P.), then the difference between the selling price and cost price is called Profit.
- ❖ **Loss** : If the selling price (S.P.) of an article is less than the cost price (C.P.), the difference between the cost price (C.P.) and the selling price (S.P.) is called Loss.
- ❖ Gain = S.P. - C.P., if S.P. > C.P.
- ❖ Loss = C.P. - S.P., if C.P. > S.P.
- ❖ Gain % = %% Loss% = $\frac{\text{Loss} \times 100}{\text{CP}}$
- ❖ S.P. = $\frac{100 + \text{Gain}\%}{100} \times \text{C.P.}$
- ❖ S.P. = $\frac{100 - \text{Gain}\%}{100} \times \text{C.P.}$
- ❖ When the selling price and gain percent are given, the
C.P. = $\frac{100}{100 + \text{Gain}\%} \times \text{S.P.}$
- ❖ When the selling price and loss percent are given then
C.P. = $\frac{100}{100 - \text{Gain}\%} \times \text{S.P.}$

Discount

Discount means reduction in the price. This reduction is always given on the marked price (M.P.) or List price (L.P.).

- ❖ When discount is offered on an article, then we calculate the selling price (S.P.) as :
S.P. = Marked Price - Discount.
- ❖ Discount = Marked price - Selling price
- ❖ Discount% = $\frac{\text{MP.} \times \text{RATE of discount}}{100}$

$$\text{S.P.} = \text{M.P.} \times \left(\frac{100 - \text{Discount}}{100} \right)\%$$

$$\text{M.P.} = \frac{100 \times \text{S.P.}}{100 - \text{Discount}\%}$$

- ❖ If the S.P. of two objects are equal and one of them is sold at x% profit and other is at x% loss then there is always a loss of = $\left(\frac{\text{Common Loss or Gain}\%}{10} \right)^2$

$$\frac{x^2}{100}\%$$

- ❖ Two successive discount of x% and y% allowed on an item are equivalent to a single discount of $\left(x + y - \frac{xy}{100} \right)\%$

- ❖ **NOTE** : This discount is always less than the sum of individual discount.

Ex.1 If the manufacture gains 10% on an article, the wholesale dealer gain 15% and the retailer 25%, then find the cost of production of a table whose retail price is Rs. 1265 ?

Sol. Let the cost of production of the table be Rs. x.
Then, 125% of 115% of 110% of x = 1265

$$\Rightarrow \frac{125}{100} \times \frac{115}{100} \times \frac{110}{100} \times x = 1265$$

$$\Rightarrow \frac{253}{160} x = 1265$$

$$\Rightarrow x = \left(\frac{1265 \times 160}{253} \right) = \text{Rs. } 800.$$

Ex.2 A man sold two flats for Rs. 675958 each. On one he gains 16% while on the other he loses 16%. How much percentage does he gain or lose in the whole transaction ?

Sol. Loss% = %%

Ex.3 A man bought toffees at 3 for a rupee. How many toffees for a rupee must he sell to gain 50% ?

Sol. C.P. of 3 toffees = Rs 1:

S.P. of 3 toffees = 150% of Rs. 1

$$= \text{Rs. } \frac{3}{2}$$

$$\text{For Rs. } \frac{3}{2} \text{ toffees sold} = 3$$

$$\text{For Rs. } 1, \text{ toffees sold} = \left(3 \times \frac{2}{3} \right) = 2$$

Ex.4 The C.P. of 21 articles is equal to S.P. of 18 article.
Find the gain or loss percent.

Sol. Let C.P. of each article be Rs. 1.
Then, C.P. of 18 articles = Rs.18,
 \therefore S.P. of 18 articles = Rs. 21.
 \therefore Gain% $\left(\frac{3}{18} \times 100 \right) \% = 16\frac{2}{3}\%$

Ex.5 By selling 33 metres of cloth, one gain the selling of 11 metres. Find the gain percent.

Sol. (S.P. of 33m) - (C.P. of 33 m) = Gain S.P. of 11 m.
 \therefore S.P. of 22 m = C. P. of 33 m.
Let C.P. of each metre be Rs. 1.
Then, C.P. of 22 m = Rs. 22.
and, S.P. of 22 m = Rs. 33.
 \therefore Gain% $= \left(\frac{11}{22} \times 100 \right) \% = 50\%$

Ex.6 A vendor bought bananas at 6 for Rs. 10 and sold them at 4 for Rs. 6. Find his gain or loss percent.

Sol. Suppose, number of bananas bought = L.C.M.(6, 4) = 12

$$\therefore \text{C.P.} = \text{Rs.} \left(\frac{10}{6} \times 12 \right) = \text{Rs.} 20;$$

$$\text{S.P.} = \text{Rs.} \left(\frac{6}{4} \times 12 \right) = \text{Rs.} 18;$$

$$\therefore \text{Loss\%} = \left(\frac{2}{20} \times 10 \right) = \text{Rs.} 10\%;$$

Ex.7 At what percentage above the C.P. must an article be marked so as to gain 33% after allowing a customer a discount of 5%.

Sol. Let C.P. = Rs. 100. Then S.P. = Rs. 133.
Let market price be Rs. x.
Then 95% of x = 133.

$$\Rightarrow \frac{95}{100} x = 133 \Rightarrow x = \left(133 \times \frac{100}{95} \right) = 140.$$

Hence, percentage above the C.P. of an article be marked as = 140 - 100 = 40%.

Ex.8 A tradesman sold an article at a loss of 20%. If the selling price had been increased by Rs. 100, there would have been a gain of 5%. What was the cost price of the article ?

Sol. Let the C.P. be Rs. x.

$$1\text{st S.P.} = 80\% \text{ of } x = \frac{80}{100}x = \frac{4x}{5}$$

$$2\text{nd S.P.} = 105\% \text{ of } x = \frac{105}{100}x = \frac{21}{20}x$$

A.T.Q.

$$\frac{21}{20}x - \frac{4}{5}x = 100$$

$$\Rightarrow x = \frac{20}{5} \times 100$$

$$\Rightarrow x = 400$$

Hence, C.P. = Rs. 400.

Ex.9 Find the single discount equivalent to a series discount of 20%, 10% and 5%.

Sol. Let market price be Rs. 100.

Then, net S.P. = 95% of 90% of 80% of Rs. 100.

$$= \text{Rs.} \left(\frac{95}{100} \times \frac{90}{100} \times \frac{80}{100} \times 100 \right) = \text{Rs.} 68.40.$$

\therefore Required discount = (100 - 68.40%) = 31.6%.

Ex.10 A man bought a horse and a carriage for Rs. 3000.

He sold the horse at a gain of 20% and the carriage at a loss of 10%, there by gaining 2% on the whole. Find the cost of the horse.

Sol. Let the C.P. of the horse be Rs. x.

Then, C.P. of the carriage = Rs. (3000 - x).

\therefore 20% of x - 10% of (3000 - x) = 2% of 3000.

$$\Rightarrow \frac{x}{5} - \frac{(3000 - x)}{10} = 60$$

$$\Rightarrow 2x - 3000 + x = 600$$

$$\Rightarrow 3x = 3600$$

$$\Rightarrow x = 1200. \text{ Hence, C.P. of the horse} = \text{Rs.} 1200$$

EXERCISE

- Sam purchased 20 dozens of toys at the rate of Rs. 375 per dozen. He sold each one of them at the rate of Rs. 33. What was his percentage profit ?
(A) 50.6% (B) 5.6%
(C) 4.6% (D) 6.5%
- A shopkeeper expect a gain of $22\frac{1}{2}\%$ on his cot race. If in a week, his sale was of Rs. 392, what was his profit ?
(A) Rs.28 (B) Rs. 72
(C) Rs. 72 (D) Rs. 88.25
- In the cost price of 12 pens is equal to the selling price of 8 pens, the gain percent is
(A) $66\frac{2}{3}\%$ (B) $33\frac{1}{3}\%$
(C) 50% (D) 25%
- In the selling price of 50 articles is equal to the cost price of 40 articles, then the loss or gain percent is :
(A) 20% loss (B) 20% gain
(C) 25% loss (D) 25% gain 44.
- Find a single discount equivalent to the series of two discounts of 15% and 4%.
(A) 20% (B) 21.6%
(C) 25% (D) 18.4%
- A man buys eggs at 2 for Rs. 1 and an equal number at 3 for Rs. 2 and sell the whole at 5 for Rs. 3. His gain or loss percent is :
(A) $2\frac{2}{7}\%$ loss (B) $3\frac{6}{7}\%$ gain
(C) $3\frac{2}{7}\%$ loss (D) $2\frac{6}{7}\%$

7. A man bought some oranges at Rs. 10 per dozen and bought the same number of oranges at Rs. 8 per dozen. He sold these oranges at Rs. 11 per dozen and gained Rs. 120. The total number of oranges bought by him was :
 (A) 30 dozens (B) 40 dozens
 (C) 50 dozens (D) 60 dozens
8. Padam purchased 30 kg of rice at the rate of Rs. 17.5/kg and another 30 kg rice at a certain rate. He mixed the two and sold the entire quantity at the rate of Rs. 18.60/kg and made 20% overall profit. At what price per kg did he purchase the lot of another 30 kg rice ?
 (A) Rs. 12.50 (B) Rs. 13.50
 (C) Rs. 14.50 (D) Rs. 15.50
9. A dishonest dealer professes to sell his goods at cost price. But he uses a false weight and thus gains $6\frac{18}{47}\%$. For a kg, he uses a weight of :
 (A) 940 gm (B) 947 gm
 (C) 953 gm (D) 960 gm
10. A man buys an article for 10% less than its value and sells it for 10% more than its value. His gain or loss percent is :
 (A) no profit, no loss (B) 20% profit
 (C) less than 20% profit (D) more than 20% profit
11. The cash difference between the selling prices of an article at a profit of 4% and 6% is Rs. 3. The ratio of the two selling prices is :
 (A) 51 : 52 (B) 52 : 53
 (C) 51 : 53 (D) 52 : 55
12. By selling an umbrella for Rs. 300, a shopkeeper gains 20%. During a clearance sale, the shopkeeper allows a discount of 10% on the marked price. His gain percent during the sale is :
 (A) 7 (B) 7.5
 (C) 8 (D) 9
13. At what price should a shopkeeper mark a ratio that costs him Rs. 1200 in order that he may offer a discount of 20% on the marked price and still make a profit of 25%
 (A) Rs. 1675 (B) Rs. 1875
 (C) Rs. 1900 (D) Rs. 2025
14. Peter bought an item at 20% discount on its original price. He sold it with 40% increase on the price he bought it. The new sale price is by what percent more than the original price ?
 (A) 7.5% (B) 8%
 (C) 10% (D) 12%
15. Varun got 30% concession on the labeled price of an article and sold it for Rs. 8750 with 25% profit on the price he bought. What was the labeled price ?
 (A) Rs. 10,000 (B) Rs. 12,000
 (C) Rs. 16,000 (D) Data inadequate
16. In the selling price of 8 articles is equal to the cost price of 10 articles, then the gain or loss percentage is
[NTSE Stage-I/RAJ./2007]
 (A) $\frac{10-8}{8} \times 100$ gain (B) $\frac{10-8}{8} \times 100$ loss
 (C) $\frac{10-8}{10} \times 100$ gain (D) $\frac{10-8}{10} \times 100$ loss
17. A sold a commodity to B with 10% profit. If B resold the same commodity to A with a loss of 10%, then A will have :
(NTSE Stage-I/Raj./2007)
 (A) 1% loss (B) 11% loss
 (C) 1% profit (D) 11% profit
18. If an article is sold for Rs. p, there is a loss of 15%. If however, the same article is sold for Rs. q, there is a profit of 15% then the ratio (q - p) : (q + p) is :
(NTSE state -II/2008)
 (A) 20 : 23 (B) 20 : 3
 (C) 3 : 20 (D) 17 : 23



RATIO PROPORTION & PARTNERSHIP

Ratio and Proportion

Ratio : If a and b ($b \neq 0$) are two quantities of the same kind, the fraction $\frac{a}{b}$ is called the ratio of a to b, we write is a : b.

Proportion : Four (non-zero) quantities of the same kind a,b,c & d are said to be in proportion if the ratio of a to b is equal to the ratio of c to d.

i.e. if $\frac{a}{b} = \frac{c}{d}$ we write is as a : b :: c : d.

- ❖ Here a,b,c & d are called first second, third & fourth proportions respectively.
- ❖ a,b,c & d are in proportion if $ad = bc$.

Continued Proportion : The (non-zero) quantities of the same kind a,b,c,d,e,f,... are said to be in continued

proportion if $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \frac{e}{f}$

Mean Proportional : If a,b, & c are in continued proportion, then a & c called first & third proportional respectively and b is called mean proportional of a and c. So, a : B :: B : c.

Then, $\frac{a}{b} = \frac{b}{c}$

$$\Rightarrow b^2 = ac$$

$$\Rightarrow b = \sqrt{ac}$$

Compound Ratio : The compound ratio of the ratios a : b, c : d & e : f is ace : bdf.

Comparison of Ratios : We say that :

$$(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$$

Duplicate Ratio : The duplicate ratio of a : b is $a^2 : b^2$.

Sub duplicate Ratio : The sub-duplicate ratio of a : b is $\sqrt{a} : \sqrt{b}$

Triplicate Ratio : The triplicate ratio of a : b is $a^3 : b^3$.

Sub triplicate Ratio : The sub-triplicate ratio of a : b is $\frac{1}{a^3} : \frac{1}{b^3}$

Invertendo : $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$

Altermendo : $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$

Componendo : $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{b} = \frac{c+d}{d}$

Dividendo : If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a-b}{b} = \frac{c-d}{d}$

Componendo and Dividendo : If $\frac{a}{b} = \frac{c}{d}$

$$\text{then, } \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

❖ If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ then each ratio is equal to $\frac{a+c+e}{b+d+f}$

Ex.1 if a : b = 5 : 9 and b : c = 4 : 7, find a : b : c.

Sol. a : b = 5 : 9 and b : c = 4 : 7

$$\text{or, } b : c \left(4 \times \frac{9}{4} \right) : \left(7 \times \frac{9}{4} \right) = 9 : \frac{63}{4}$$

$$\Rightarrow a : b : c = 5 : 9 : \frac{63}{4} = 20 : 36 : 63.$$

Ex.2 Find the fourth proportional of 5,8,15.

Sol. Let, the fourth proportional is x.

So, 5 : 8 :: 15 : x

$$\Rightarrow \frac{5}{8} = \frac{15}{x}$$

$$\Rightarrow 5 \times x = 15 \times 8$$

$$\Rightarrow x = \frac{15 \times 8}{5} = 24$$

Ex.3 if Rs. 782 be divided into three parts, proportional

to $\frac{1}{2} : \frac{2}{3} : \frac{2}{4}$, then find the first part.

Sol. Given ratio = 12 : $\frac{2}{3} : \frac{3}{4} = 6 : 8 : 9f$

$$\therefore \text{1st part} = \text{Rs. } \left(782 \times \frac{6}{23} \right) = 204.$$

Ex.4 A bag contains 50 p, 25 p and 10 p coins in the ratio 5 : 9 : 4, amounting to Rs. 206. Find the number of coins of each type.

Sol. Let the number of 50 p, 25 p and 10 p coins be 5x, 9x and 4x respectively.

$$\text{Then, } \frac{5x}{2} + \frac{9x}{4} + \frac{4x}{10} = 206$$

$$\Rightarrow 50x + 45x + 8x = 4120$$

$$\Rightarrow 10x = 4120$$

$$\Rightarrow x = 40$$

$$\therefore \text{Number of 50 p coins} = (5 \times 40) = 200;$$

$$\text{Number of 25 p coins} = (9 \times 40) = 360;$$

$$\text{Number of 10 p coins} = (4 \times 40) = 160.$$

Partnership

Partnership : When two or more than two persons run a business jointly, they are called partners and the deal is known as partnership.

Ratio of Division of Gains :

(i) When investments of all the partners are for the same time, the gain or loss is distributed among the partners in the ratio of their investments.

- ❖ Suppose A and B invest Rs. x and Rs. y respectively for a year in business, then at the end of the year :

(A's share of profit) : (B's share of profit) = x : y.

(ii) When investments are for different time periods, then equivalent capitals are calculated for a unit of time by taking (capital × number of units of time). Now, gain or loss is divided in the ratio of these capitals.

- ❖ Suppose A invests Rs. x for t_1 months and B invests Rs. y for t_2 months, then

$$\frac{\text{A's share in profit}}{\text{B's share in profit}}$$

$$\frac{\text{A's amount} \times \text{A's time of investig of money}}{\text{B's amount} \times \text{B's time of investing of money}}$$

$$\Rightarrow \frac{\text{A's share in profit}}{\text{B's share in profit}} = \frac{y \times t_1}{x \times t_2}$$

Working and Sleeping Partners : A partner who manages the business is known as a working partner and the one who simply invests the money is a sleeping partner.

- Ex.5** A and B invested Rs. 3600 Rs. 4800 respectively to open a shop. At the end of the year B's profit was Rs. 1208. Find A's profit.

Sol. Profit sharing ratio = 3600 : 4800 = 3 : 4

$$\frac{\text{Profit of A}}{\text{Profit of B}} = \frac{3}{4}$$

$$\therefore \text{Profit of A} = \frac{3}{4} \text{ Profit of B}$$

$$\Rightarrow \text{Profit of A} = \frac{3}{4} \times 1208 = \text{Rs. } 906$$

- Ex.6** A starts business with Rs. 3500 and after 5 months, B joining with A as his partner. After a year, the profit is divided in the ratio 2 : 3. What is B's contribution in the capital ?

Sol. Let B's capital be Rs. x .

$$\text{Then, } \frac{3500 \times 12}{7x} = \frac{2}{3}$$

$$\Rightarrow 14x = 126000$$

$$\Rightarrow x = 9000.$$

EXERCISE

- The third proportional of 0.36 and 0.48 is :
(A) 0.64 (B) 0.1728
(C) 0.42 (D) 0.94
- If $2A = 3B$ and $4B = 5C$, then A : C is :
(A) 4 : 3 (B) 8 : 15
(C) 15 : 8 (D) 3 : 4
- If $x : y = 5 : 2$, then $(8x + 9y) : (8x + 2y)$ is :
(A) 22 : 29 (B) 26 : 61
(C) 29 : 22 (D) 61 : 26
- Two numbers are in the ratio 3 : 5. If 9 is subtracted from each, the new numbers are in the ratio 12 : 23. The smaller number is :
(A) 27 (B) 33
(C) 49 (D) 55
- The ratio of three numbers is 3 : 4 : 7 and their product is 18144. The numbers are :
(A) 9,12,21 (B) 15,20,25
(C) 18,24,42 (D) None of these
- A sum of Rs. 1300 is divided amongst P,Q, R and S such that :
$$\frac{\text{P's share}}{\text{Q's share}} = \frac{\text{Q's share}}{\text{R's share}} = \frac{\text{R's share}}{\text{S's share}} = \frac{2}{3}$$

Then P's share is :
(A) Rs. 140 (B) Rs. 160
(C) Rs. 240 (D) Rs. 320
- What least common number must be subtracted from each fo The numbers 14, 17, 34 and 42 so that the remainders may be proportional ?
(A) 0 (B) 1
(C) 2 (D) 7
- Gold is 19 times as heavy as water and copper is 9 times as heavy as water. In what ratio should copper and gold be mixed to get and alloy 15 times as heavy as water ?
(A) 1 : 1 (B) 2 : 3
(C) 1 : 2 (D) 3 : 2
- The speeds of three cars are in the ratio 5 : 4 : 6. The ratio between the time taken by them to travel the same distance is :
(A) 5 : 4 : 6 (B) 6 : 4 : 5
(C) 10 : 12 : 15 (D) 12 : 15 : 10
- Monthly incomes of A and B are in the ratio of 4 : 3 and their savings are in ratio of 3 : 2. If the expenditure of each will be Rs. 600, what will be the monthly income of B ?
(A) 2400 (B) 1170
(C) 1500 (D) 1800
- In a bag there are coins of 25p, 10p and 5p in the ratio of 1 : 2 : 3. If there are Rs. 30 in all, how many 5p coins are there ?
(A) 50 (B) 100
(C) 150 (D) 200
- Ramesh started a business by investing Rs. 25000. 3 months later Mahesh joined the business by investing Rs. 2500. AT the end of the year Ramesh got Rs. 1000 more than Mahesh out of the profit. Find the total profit.
(A) Rs. 6000 (B) Rs. 6500.
(C) Rs. 7000 (D) Rs. 7500
- A started a business by investing Rs. 8000. After x months B joined the business by investing Rs. 12,000. If the profit earned is equal by both at the end of the year then find the value of x .
(A) 3 months (B) 4 months
(C) 6 months (D) 8 months
- A and B started a business with initial investments in the ratio 14 : 15 and their annual profits were in the ratio 7 : 6. If a invested the money for 10 months, for how many months did B invest his money ?
(A) 6 (B) 7
(C) 8 (D) 9
- A, B and C enter into a partnership with a capital in which A's contribution is Rs. 10,000. If out of a total profit of Rs. 1000, A gets Rs. 500 and B gets Rs. 300, then C's capital is :
(A) Rs. 4000 (B) Rs. 5000
(C) Rs. 6000 (D) Rs. 9000

TIME, SPEED AND DISTANCE

Time Speed & Distance

- (i) $\text{Speed} = \left(\frac{\text{Distance}}{\text{Time}} \right)$, $\text{Time} = \left(\frac{\text{Distance}}{\text{Speed}} \right)$,
 $\text{Distance} = (\text{Speed} \times \text{Time})$
- (ii) $X \text{ km/hr} = \left(x \times \frac{5}{18} \right) \text{ m/sec}$
- (iii) $X \text{ m/sec} = \left(x \times \frac{18}{5} \right) \text{ km/hr}$
- (iv) If the ratio of the speeds of A and B is $a : b$, then the ratio of the times taken by them to cover the same distance is $\frac{1}{a} : \frac{1}{b}$ or $b : a$
- (v) Suppose a man covers a certain distance at $x \text{ km/hr}$ and an equal distance at $y \text{ km/hr}$. then , then average speed during the whole journey is $\left(\frac{2xy}{x+y} \right) \text{ km/hr}$.

Problems on Trains

- (i) Time taken by a train of length 'a' metres to pass a pole or a standing man or a signal post is equal to the time taken by the train to cover 'a' meters.
- (ii) Time taken by a train of length 'a' metres to pass a stationary object of length 'b' metres is the time taken by the train to cover $(a+b)$ meters.
- (iii) Suppose two trains or two bodies are moving in the same direction at $u \text{ m/s}$ and $v \text{ m/s}$, where $u > v$, then their relative speed = $(u-v) \text{ m/s}$.
- (iv) Suppose two trains or two bodies are moving in opposite direction at $u \text{ m/s}$ and $v \text{ m/s}$, where $u > v$, then their relative speed = $(u+v) \text{ m/s}$.
- (v) If two trains of length 'a' metres and 'b' metres are moving in opposite directions at $u \text{ m/s}$ and $v \text{ m/s}$, then time taken by the trains to cross each other $= \left(\frac{a+b}{u+v} \right) \text{ sec}$
- (vi) If two trains of length 'a' metres and 'b' metres are moving in the same direction at $u \text{ m/s}$ and $v \text{ m/s}$ then the time taken by the faster train to cross the slower train = $\frac{(a+b)}{(u-v)} \text{ sec}$.
- (vii) If two trains (or bodies) start at the same time from points A and B towards each other and after crossing they take 'a' and 'b' sec in reaching B and A respectively, then 'a' and 'b' sec in reaching B and A

respectively, the $(A's \text{ speed}) : (B's \text{ speed}) = (\sqrt{b} : \sqrt{a})$

EXERCISE

- Two trains running in the same direction at 40 km/hr and 22 km/hr completely pass one another in 1 minute. If the length of the 1st train is 125 m, then what will be length of 2nd train.
 (A) 175 m (B) 180 m
 (C) 170 m (D) 185 m
- A train overtakes two persons walking along a railway track. The first one walks at 4.5 km/hr . The other one walks at 5.4 km/hr . The train needs 8.4 and 8.5 seconds respectively to overtake them. What is the speed of the train if both the persons are walking in the same direction as the train?
 (A) 81 km/hr (B) 72 km/hr
 (C) 78 km/hr (D) 99 km/hr
- A train traveling at a speed of 45 km/hr reaches Chennai from Howrah (A distance of 180 km). On the return journey its speed is 36 km/hr . Find the average speed of train in two journeys.
 (A) 40.5 km/hr (B) 37 km/hr
 (C) 39 km/hr (D) 40 km/hr
- Naresh Kumar drives his car to his office with the speed of 40 km per hour and returns along the same route with the speed of 60 km per hour . His average speed for the entire round trip is
 (A) 50 km/hr (B) 48 km/hr
 (C) 45 km/hr (D) None of these
- A dog at point P goes in pursuit of a fox 40 m leap of the fox. If the dog makes two leap of m against 1m long leap of the fox. If the dog makes two leaps to the fox' three at what distance from P will the dog catch up with the fox?
 (A) 150 m (B) 160 m
 (C) 105 m (D) 120 m
- It takes 12 hrs for a bus to cover the distance between the cities, X and Y. A bus leaves X for Y at 6 p.m., and another bus leaves Y for X at 8 p.m. At what time will same the two buses meet? (Assume both travel with same speed)
 (A) 1 a.m. (B) 2 a.m.
 (C) 4 a.m. (D) 6 a.m.

7. A thief robs a house at 12 midnight, and as soon as he leaves the house, the house owner realises of the robbery in the house, the After 10 minutes he rings the alert alarm, and the security guards of the house speed of the thief is 30 km/hr and that of the security guards is 20 km/hr, what time will the guards catch the thief?
 (A) 00.30 hrs (B) 00.40 hrs
 (C) cannot be determined (D) Never catch the thief
8. Vijay and Shivku start simultaneously from the opposite ends of a pool which is 50 m long. They pass each other, reach the respective end immediately turn back now they meet at a distance of 15 m from where vijay started, 10 s after the start. Find the speed of Shivku.
 (A) 6.5 m/s (B) 7.5 m/s
 (C) 8.5 m/s (D) 5 m/s
9. A man is traveling by car at the rate of 40 km/hr. After every 80 km, he rests for 20 min. How long will he take to cover a distance of 240 km?
 (A) 6 hr 40 min (B) 6 hr
 (C) 6 hr 20 min (D) 7 hr
10. A motor car starts with the speed of 70 km/hr with its speed increasing every two hours by 10 kmph. In how many hours will it cover 345 kms?
 (A) $2\frac{1}{4}$ hrs (B) 4 hrs 5 min
 (C) $4\frac{1}{2}$ hrs (D) can't determined
11. Anu left for city A from city B at 5.20 a.m. She traveled at the speed of 80 km/hr for 2 hours 15 minutes. After that the speed was reduced to 60 km/hr. If the distance between two cities is 350 kms, at what time did Anu reach city B?
 (A) 9.20 a.m. (B) 9.25 a.m.
 (C) 9.35 a.m. (D) none of these
12. If a person has a speed of 40 km/hr he reaches the office 5 min late and if he increases his speed to 50 km/hr, he reaches the office 3 min early. Calculate the distance to be covered
 (A) 18 km (B) 26.6
 (C) 30 km (D) 36 km
13. It takes 4 hr to go from Kolkata to Kharagpur. A truck of Durga Transport leaves Kolkata for Kharagpur every 30 min starting at 6.00 a.m. and travel at the speed of the trucks, how many such trucks would you meet till you reach Kolkata?
 (A) 8 (B) 9
 (C) 16 (D) 17
14. Walking $\frac{7}{6}$ th of his usual rate, a boy reaches his school 4 minutes early. Find his usual time to reach the school.
 (A) 40 min (B) 32 min
 (C) 28 min (D) 48 min
15. A man travels 100 kms in 5 hrs and another 200 kms in 15 hrs. Find the average speed.
 (A) 15 km/hr (B) 10 km/hr
 (C) 20 km/hr (D) None of these
16. A monkey ascends a greased pole of 12 mts height. It ascends 2 m in the first minute and slips and slips down 1 m in the alternate minute. In Which minute will it reach the top?
 (A) 11th (B) 12th
 (C) 21st (D) 22nd
17. A train running at a speed of 52 km/hr in the same direction. Find the length of the train.
 (A) 100 m (B) 150 m
 (C) 300 m (D) 200 m
18. A train 140 m long is running at 60 km/hr. In how much time will it pass a tunnel 260 m long?
 (A) 18 sec (B) 24 sec
 (C) 30 sec (D) 15 sec
19. A train crosses a telegraph post in 8 sec and a bridge 200 m long in 24 sec. Find the speed of the train.
 (A) 32 km/hr (B) 40 km/hr
 (C) 45 km/hr (D) None of these
20. Excluding stoppages, the speed of a train is 45 km/hr. and including stoppages it is 36 km/hr. For how many minutes does the train stop per hour?
 (A) 12 min (B) 10 min
 (C) 8 min (D) None of these
21. A 400 m long train is running at the speed of 60 km per hour. It crosses a bridge of length 800 m in –
 (NTSE Stage-1/Raj./2007)
 (A) $6\frac{2}{3}$ seconds (B) 20 seconds
 (C) 2 seconds (D) 72 seconds

WORK AND TIME

Unitary Method and Complex ratio

- A.** Direct Proportion : Two quantities are said to be directly proportional , if on the increase (or decrease) of the one, the other increases (or decreases) to the same extent.
- ❖ Cost is directly proportional to the number of articles. (More Articles, More Cost)
 - ❖ Work done is directly proportional to the number of men working on it (More Men ,More Work)
- B.** Indirect Proportion : Two quantities are said to be indirectly proportional, if on the increase of the one, the other decreases to the same extent and vice – versa .
- ❖ The time taken by a car in covering a certain distance is inversely, proportional to the speed of the car .(More speed, Less is the time taken to cover a distance).
 - ❖ Time taken to finish a work is inversely proportional to the number of persons working at it .(More persons, Less is the time taken to finish a job).

Work and Time

- (i) If A can do a piece of work in m days, then :
A's 1day work is $= \frac{1}{m}$.
- (ii) If B's 1day work $= \frac{1}{n}$, then :
B can finish the complete work in n days.
- (iii) If A is thrice as good a workman as B, then :
Ratio of work done by A and B is = 3:1
Ratio of times taken by A and B to finish a work is = 1:3

Pipes and Cisterns

- ❖ Inlet Pipe : A pipe connected with a tank or a cistem or a reservoir , that fills it , is known as an inlet.
Outlet Pipe : A pipe connected with a tank or a cistem or a reservoir, emptying it , is known as an outlet.
- (i) If a pipe can fill a tank in x hours, then :
Part filled in 1 hour $= \frac{1}{x}$
- (ii) If a pipe can empty a full tank in y hours, then,
Part emptied in 1 hour $= \frac{1}{y}$.
- (iii) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $y > x$). Then on opening both the pipes, the net part filled in 1 hour
- $$= \left(\frac{1}{x} - \frac{1}{y} \right)$$

- (iv) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y houres (where $x > y$), then on opening both the pipes , the net part emptied in 1

$$\text{hour} = \left(\frac{1}{y} - \frac{1}{x} \right)$$

EXERCISE

1. 24 men can complete a given job in 40 days. Then find the number of men required to complete job in 32-days.

(A) 25 men	(B) 30 men
(C) 32 men	(D) 35 men
2. If 5 men can cane 5 chairs in 5 hours, then 1 man shall cane 1 chair in how many hours ?

(A) 1hour	(B)15 hour
(C) 5 hours	(D) 10 hours
3. If a workers can dig 20 m long ditch in 6 days, find out the par dug by 2 workers in a day (in metre)

(A) $1\frac{2}{3}$	(B) $3\frac{2}{4}$
(C) $3\frac{2}{3}$	(D) $1\frac{3}{2}$
4. 25 men were employed to do a place of work in some days. After 15 days, 10 more men were engaged and the work was finished in 23 days. In what time could they finish the work if extra men were not employed .

(A) 24 days	(B) 24.6 days
(C) 26.2 days	(D) None of these
5. If 12 men or 18 women can reap a field in 7 days , in what time can 4 men & 8 women reap the same field.

(A)8 days	(B) 9 days
(C) 10days	(D) None of these
6. A can do a job in 3 days, and B can do the same job in 6 days . How long will if take them if they work together ?

(A) 1 day	(B) 2 day
(C) 3 day	(D) 4 day
7. A and B together can complete a work in 12 days. A alone can complete it in 20 days , If B now does the work for half a day daily then in how many days A and B together will complete the work ?

(A) 15 days	(B) 20 days
(C) 45 days	(D) 30 days

8. Harpal is thrice as good a workman as Kewal and takes. Then Kewal can do that work in:
 (A) 8 days (B) 12 days
 (C) 13 days (D) 15 days
9. A can do a piece of work in 9 days and B in 18 days. They began the work together but 3 days before the completion of work, A leaves. The time taken to complete the work is :
 (A) 7days (B) 5days
 (C) 8 days (D) 11 days
10. A,B and C can complete a working alone in 10, 15 and 20 days respectively. If all of them work together to complete the work, what fraction of the work would have done by B ?
 (A) $\frac{4}{13}$ (B) $\frac{1}{2}$
 (C) $\frac{1}{3}$ (D) $\frac{6}{13}$
11. A,B and C can complete a piece of work in 24,6 and 12 days respectively. Working together, they will complete the same work in :
 (A) $\frac{1}{24}$ day (B) $\frac{7}{24}$ days
 (C) $3\frac{3}{7}$ days (D) 4 days
12. A,B and C together can finish a piece of work in 4 days. A alone can do it in 9 days and B alone 18 days. How days will be taken by c to do it alone.
 (A) 15days (B) 14 days
 (C) 12 days (D) 10 days
13. A and B can do a work in 8 days, B and C can do the same work in 12 days. A,B and C together can finish it in 6 days. A and C together will do it in :
 (A) 4 days (B) 6 days
 (C) 8days (D) 12 days
14. A and do a place of work in 4 hours; B and C together can do it in 3 hours, while A and C together can do it in 2 hours. How long will B alone take to do it ?
 (A) 8 hours (B) 12 hours
 (C) 15 hours (D) 24 hours
15. A takes twice as much times B and thrice as much time of C to finish a piece of work. Working together, they can finish the work in 2 days. B can do the work alone in :
 (A) 4 days (B) 6 days
 (C) 8 days (D) 12 days
16. Two pipes M&N can fill a cistem in 12 & 16 hrs. respectively. If both are opened together, then after how many hrs N should be closed so that the tank is full in 9 hrs.
 (A) 3hrs (B) hrs
 (C) 4hrs (D) None of these
17. Three pipes A,B & C can fill a tank in 6 hrs. After working at it together for 2 hrs, C is closed and A and B can fill it in 7 hrs, C is closed and A and B can fill it in 7 hrs. The number of hours taken by C alone to fill the cistem, is :
 (A) 14 hours (B) 15 hours
 (C) 16 hours (D) None of these
18. 4 pipes can fill a tank in 15,20,30& 60 hours, respectively. The first was opened at 6 a.m. second at 7 a.m. third at a.m. and fourth at 9 a.m. when will the tank be full ?
 (A) 12.30 pm (B) 1 pm
 (C) 2 pm (D) None of these
19. Taps A and B can fill a tank in 12 min and 15 min respectively. If both are opened and A is closed after 3 min, how long will it take for B to fill the tank ?
 (A) 7min 45 s (B) 7 min 15 s
 (C) 8 min 5 s (D) 8 min 15 s
20. A leak in the bottom of a tank can empty it in 6 hr. A pipe fills the tank at 4 L/min. When the tank is full, the inlet is opened, but due to the leak the tank is emptied in 8 hr. What is the capacity of the tank ?
 (A) 5,260 L (B) 5,760 L
 (C) 5,846 L (D) 6.970 L
21. A can do $\frac{3}{8}$ part of a work in 9 hrs, B can do $\frac{1}{4}$ part of the same work in 4 hrs. Both work together and complete the work in : **(NTSE Stage-1/Raj./2007)**
 (A) $\frac{48}{5}$ hrs (B) $\frac{5}{48}$ hrs
 (C) $\frac{35}{8}$ hrs (D) $\frac{8}{35}$ hrs

S.I.&C.I.

Simple interest

Principal : Whenever we borrow money from some lending source such as bank or a financial institution, we have to pay some extra money to the service provider, which depends upon the sum borrowed. This extra money is called the interest. The borrowed money is called principal.

- ❖ The rate of interest is taken as “percent per annum” which means “per Rs.100 per year” i.e. a rate of ‘10% per annum’, means Rs 10 on RS. 100 for 1 year.
Amount : When interest is calculated simply on the original principal, it is known as Simple Interest. When the interest for a specific period is added to the principal, then the sum is called the amount.
- ❖ $S.I = \frac{P \times R \times T}{100}$
- ❖ $P = \frac{100 \times S.I}{R \times T}$
- ❖ $R = \frac{100 \times S.I}{P \times T}$
- ❖ $T = \frac{100 \times S.I}{R \times P}$

Ex.1 Find the rate of interest at which a sum becomes four times of itself in 15 years at S.I.

Sol. Let sum is P then,
 $\Rightarrow 3P = \frac{P \times R \times 15}{100}$
 $\Rightarrow R = 20\%$

Ex.2 How long will it take a sum of money invested at 5% p.a.S.I. to increase its value by 40% ?

Sol. Let sum is P then S.I. is $\frac{2}{5} P$
 $\Rightarrow \frac{2}{5} P = \frac{P \times 5 \times T}{100}$
 $\Rightarrow T = 8 \text{ years.}$

Ex.3 In what time a sum of Rs.2700 amounts to Rs. 3240 at a rate of $6\frac{2}{3}\%$ at S.I.?

Sol Given .
 Principal = Rs. 2700
 Amount – Rs. 3240
 Interest = Rs. (3240-2700) =Rs. 540
 Rate = $6\frac{2}{3}\%$
 $\therefore \text{ Simple Interest} = \frac{PRT}{100}$

$$\Rightarrow 540 = 2700 \times \frac{20}{3 \times 100} \times T$$

$$\Rightarrow T \times = \frac{20}{300} \times 2700 = 18$$

$$\Rightarrow T = 3 \text{ years}$$

Hence, the required time is 3 years.

Compound Interest

- ❖ If the interest earned of a specific period is added to the principal for calculation the interest for the next period and so. On then such calculated interest is called Compound Interest (C.I).
Important Formulae : If A is the amount, P is the principal, R% is the rate of interest compounded annually and n is the number of intervals, then :

$$A = P \left(1 + \frac{R}{100} \right)^n$$

$$\text{and, C.I.} = A - P = P \left[\left(1 + \frac{R}{100} \right)^n - 1 \right]$$

- ❖ If compound interest reckoned half – yearly, then
Rate = $\frac{R}{2}$ % per half year and Time – 2n half years.

$$\text{So, } A = \left(1 + \frac{R}{2 \times 100} \right)^{2n}$$

- ❖ If compound interest reckoned quarterly, then
Rate = $\frac{R}{4}$ % per quarter, Time = 4n quarters.

$$\text{So, } A = P \left(1 + \frac{R}{4 \times 100} \right)^{4n}$$

- ❖ Let P be the principal and the rate of interest be R% annum. If the interest is compounded K-times in a year, then the amount A and the compound interest C.I. at the end of n years are given by :

$$A = P \left(1 + \frac{R}{100k} \right)^{nk}, \text{ and}$$

$$\text{C.I.} = A - P = P \left\{ \left(1 + \frac{R}{100k} \right)^{nk} - 1 \right\} \text{ respectively.}$$

- ❖ Let P be the principal and the rate of interest be R₁ % for first year, R₂ % for the R_n% year. Then, the amount A and the compound interest C.I. at the end of n years are given by :

$$A = P \left(1 + \frac{R_1}{100} \right) \left(1 + \frac{R_2}{100} \right) \dots \left(1 + \frac{R_n}{100} \right)$$

$$\& \text{ C.I.} = A - P = P \left[\left(1 + \frac{R_1}{100} \right) \left(1 + \frac{R_2}{100} \right) \dots \left(1 + \frac{R_n}{100} \right) - 1 \right]$$

- ❖ Let P be the principal and the rate of interest be R % per annum. If the interest is compounded annually but time is the fraction of a years, say $5\frac{1}{4}$ years, then amount A is given by :

$$A = P \left(1 + \frac{R}{100} \right)^5 \left(1 + \frac{\frac{R}{4}}{100} \right) \text{ and C.I.} = A - P$$

Ex.4 Find the amount of Rs.8000 for 3 years , compounded annually at 5% per annum. Also , find the compound interest.

Sol. Here, P=Rs 8000 .R=5% per annum and n=3 years.

Using the formula $A = P \left(1 + \frac{R}{100} \right)^n$,we get

$$\text{Amount at the 3 year} = \text{Rs.} \left\{ 8000 \times \left(1 + \frac{5}{100} \right)^3 \right\}$$

$$= \text{Rs} \left(8000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} \right) = \text{Rs.} 9261.$$

Ex.5 At what rate percent per annum will a sum of Rs. 2000 amount to RS 2205 in years, compounded annually ?

Sol. Let the required rate be R % per annum.

Here, A =Rs 2205 , P = Rs2000

And n=2 years.

Using the formula $A = P \left(1 + \frac{R}{100} \right)^n$. We get

$$2205 = 2000 \times \left(1 + \frac{R}{100} \right)^2$$

$$\Rightarrow \left(1 + \frac{R}{100} \right)^2 = \frac{2205}{2000}$$

$$\Rightarrow \left(1 + \frac{R}{100} \right)^2 = \frac{441}{400}$$

$$\Rightarrow \left(1 + \frac{R}{100} \right)^2 = \left(\frac{21}{20} \right)^2$$

$$\Rightarrow \left(1 + \frac{R}{100} \right) = \frac{21}{20}$$

$$\Rightarrow \frac{R}{100} = \left(\frac{21}{20} - 1 \right) = \frac{1}{20}$$

$$\Rightarrow R = \left(100 \times \frac{1}{20} \right) = 5$$

Hence, the required rate of interest is 5% per annum.

Application Of Compound Interest :

- ❖ Let P be the population of a city or town at the beginning of a certain year and the population grows at a constant rate of E% per annum, then

$$\text{Population after n years} = \left(1 + \frac{R}{100} \right)^n \times P$$

- ❖ Let P be the population of a city or a town at the beginning of a certain year. If the population grows of the sate of $R_1\%$ during first year and $R_2\%$ during second year , then

$$\text{Population after 2 years} = P \left(1 + \frac{R_1}{100} \right) \times \left(1 + \frac{R_2}{100} \right)$$

This formula may be extended for more then 2 the value V_n at the end of n years is given by

- ❖ If V_0 is the value of an article at a certain time and R% per annum is the rate of depreciation , then the value V_n at the end of n years is given by

$$V_n = V_0 \left(1 - \frac{R}{100} \right)^n$$

- ❖ If V_0 is the value of an article at a certain time and the rate of depreciation is $R_1\%$ fir first n_1 years , $R_2\%$ for next n_2 years and so on and $R_k\%$ for the last n_k years, then the value at the end of $n_1 + n_2 + \dots + n_k$ years is given by

$$V = V_0 \left(1 + \frac{R_1}{100} \right)^{n_1} \left(1 - \frac{R_2}{100} \right)^{n_2} \dots \left(1 - \frac{R_k}{100} \right)^{n_k}$$

Ex.6 The present population of a town is 25000. It grows at the rate of 4% , 5% and 8% during the first year, second year and third year respectively. Find its population after 3 years,

Sol, Population after 3 years.

$$= 25000 \left[1 + \frac{4}{100} \right] \left[1 + \frac{5}{100} \right] \left[1 + \frac{8}{100} \right]$$

$$= 25000 \times \frac{26}{25} \times \frac{21}{20} \times \frac{27}{25} = 29484$$

Hence, the population after 3 years = 29484.

EXERCISE

- A man earn Rs. 450 as an interest in 2 yrs on a certain sum invested with company at the rate of 12% per annum. Find sum invested by the man in the company.
(A) Rs.1875 (B) RS.1830
(C) Rs.1825 (D) Rs.1810
- In what time will Rs 64000 amount to Rs 68921 at 5% per annum, interest being compounded half- yearly ?
(A) $1\frac{2}{3}$ (B) $2\frac{1}{2}$ (C) $1\frac{1}{2}$ (D) $1\frac{1}{4}$
- A sum of money doubles itself ant compound interest in 15 years. In how many will it become eight times :
(A) 35 years (B) 40years
(C) 42 years (D) 45 years
- A sum of money placed at compound interest doubles itself in 5 years. If will amount to eight times itself at the same rate of interest in:
(A) 7years (B) 10years
(C) 15years (D) 20years

5. If the difference between the C.I and S.I at the end of 2 years is Rs . 100 what is the principal ? Rate is 5% per annum is both the cases . Assume same principal for both the cases.
 (A) Rs.50,000 (B) Rs.40,000
 (C) Rs.10,000 (D) None of these
6. The compound interest on RS 5000 at 4% per annum for 2 years compounded annually is :
 (A) Rs.804 (B) Rs.708
 (C) Rs.408 (D) Rs.5408
7. Find the compound interest on Rs 15625 for 9 months, at 16% per annum, compounded quarterly.
 (A) Rs.1250 (B) Rs.1651
 (C) Rs.1951 (D) Rs.2651
8. Difference between C.I.& S.I in a certain amount at 10% per annum for 2yrs compound annually is Rs .282. Find the principal.
 (A) RS.28210 (B) Rs.28120
 (C) Rs.28200 (D) None
9. If an amount is kept at S.I , It earns an interest of Rs.600 in first two years but when kept at C.I., if earns an interest of Rs .660 in the same period. The rate of interest will be :
 (A) 10% (B) 20%
 (C) 30% (D) None of these
10. The population of a village in the year 2000 was 10,000 . In the year 2001& 2002 percentage increase in population was 5%& 10% respectively. The population of village in the year 2002 will be :
 (A) $10000 \times \left(1 + \frac{5}{100}\right) \left(1 + \frac{10}{100}\right)$
 (B) $1000 \times \left(1 - \frac{5}{100}\right) \left(1 - \frac{10}{100}\right)$
 (C) $1000 \times \left(1 - \frac{5}{100}\right) \left(1 + \frac{10}{100}\right)$
 (D) $1000 \times \left(1 + \frac{5}{100}\right) \left(1 - \frac{10}{100}\right)$
11. A bacteria reproduces at the rate of 50% in every 15 min. In how much time will it be $3\frac{3}{8}$ times of itself ?
 (A) 105 min (B) 45min
 (C) 75 min (D) 50min
12. The population of a town increases 20% annually. What is the population after two years if present population is 2500 ?
 (A) 3250 (B) 3500
 (C) 3600 (D) 4000
13. What will be the ratio of simple interest earned by certainly amount at the same rate of interest for 6 years and that for 9 years ?
 (A) 1:3 (B) 1:4
 (C) Data (D) None of these
14. Rs. 300 amounts lent out at simple interest is increased by 1% then in same time the amount will be :
 (A) Rs.500 (B) RS.372
 (C) Rs.312 (D) Rs. 364
15. A sum of money lent out at simple interest amounts to Rs. 720 after 2 years and to Rs . 1020 after a further period of 5years . The sum is :
 (A) Rs. 500 (B) Rs.620
 (C) Rs.502 (D) Rs.501
16. If the compound interest on a certain sum of money for 2 years at 4% per annum is Rs. 102, then the simple interest on the same sum of money at the same rate and for the same period is
(NTSE Stage-1/Raj/2007)
 (A) Rs.99 (B) Rs.100
 (C) Rs.101 (D) Rs. 102
17. In what time a capital becomes five times at the interest rate of 10%. **(NTSE Stage-1/Raj/2007)**
 (A) 10years (B) 30years
 (C) 40 years (D) 50 years
18. An amount at certain rate of compound interest be – comes Rs. 700 in 3 years and
(NTSE Stage-1/Raj/2008)
 (A) 7% (B) 10%
 (C) 8% (D) 10.5%
19. A borrowed Rs . 500 at the rate of 5% annum and Rs. 1000 at the rate of 4% per annum on simple interest from B on the same day. Under conditions that the loan and interest will be paid when the amount in both cases together will be Rs. 2020. How many years will it take to repay the loans ? **(NTSE Stage-1/Raj/2008)**
 (A) 6 (B) 8
 (C) 10 (D) 12
20. A borrowed a sum of money for 3 years at the rate of 5% simple interest from B. He then lent that money to C for the same time and the same rate at compound interest. If A gained Rs .122 , after paying back to B, then the sum borrowed by A was :
(NTSE Stage-1/Raj/2008)
 (A) Rs.16000 (B) Rs. 12000
 (C) Rs.8000 (D) Rs.4000
21. One third of a certain sum is invested at 3% per annum ; one sixth of the sum is invested at 6% per annum and the remaining sum at 8% per annum. simple interest from all these investments is Rs .600, the original sum is : **(NTSE Stage-1/Raj/2007)**
 (A) Rs.6000 (B) Rs.6666
 (C) Rs.7500 (D) Rs. 10000

AVERAGE

Average

(i) Average = $\left(\frac{\text{Sum of observation}}{\text{Number of observation}} \right)$

(ii) Suppose aman covers a certain distance at x kmph and an equal distance at y kmph. Then the average speed during the whole journey is $\left(\frac{2x}{yx+y} \right)$ kmph.

EX.1 Find the average of all prime numbers between 30 and 50.

Sol There are five prime number between 30 and 50. They are 31,37,41,43, and 47

$$\begin{aligned} \therefore \text{ Required average} &= \left(\frac{31+37+41+43+47}{5} \right) \\ &= \frac{199}{5} = 39.8 \end{aligned}$$

EX.2 Find the average of first 20 multiples of 7.

Sol. Required average = $\frac{7(1+2+3...+20)}{20}$

$$= \left(\frac{7 \times 20 \times 21}{20 \times} \right) = \left(\frac{147}{2} \right) = 73.5$$

EX.3 The average of 25 results is 18 average of first twelve of them is 14 and of last twelve is 17. Find the thirteenth result.

Sol. Clearly, thirteenth result = (sum of 25 results) - (sum of 24 results)
 $= [(18 \times 25) - (14 \times 12) + (17 \times 12)]$
 $= 450 - (168 + 204) = 450 - 372 = 78.$

EX.4 The average age of a class of 39 students is 15 years. If the age of the teacher be included, then the average increased by 3 months. Find the age of the teacher .

Sol. Total age of 39 persons = (39×15) years = 585 years.

Average age of 40 persons = 15 years 3 months = $\frac{61}{4}$ years.

Total age of teacher = $\left(\frac{61}{4} \times 40 \right)$ years = 610 years.

\therefore Age of the teacher = $(610 - 585)$ years = 25 years.

EX.5 A batsman makes a score of 87 runs in the 17th inning and thus increases his average by 3. Find his average after 17 th inning .

Sol. Let the average after 17 thinning = x.

Then , average after 16th inning = $(x-3)$

$\therefore 16(x-3) + 87 = 17x$ or $x = (87-48) = 39.$

Ex.6 A pupil's marks were wrongly entered as 83 instead of 6.3 Due to that the average marks for the class increased by half. Find he number of pupils in class.

Sol. Let there be x pupils in the class.

$$\text{Total increase in marks} = \left(x \times \frac{1}{2} \right) = \frac{x}{2}$$

$$\therefore \frac{x}{2} = (83 - 63) \Rightarrow \frac{x}{2} = 20 \Rightarrow x = 40$$

EXERCISE

1. The average of 2,7,6 and x is 5 and the average of 18, 1,6,x and y is 10. What is the value of y ?
 (A) 5 (B) 10
 (C) 20 (D) 30
2. The average of first 50 natural numbers is :
 (A) 12.2 (B) 21.15
 (C) 25 (D) 25.5
3. The average of four consecutive even number '27. Find the largest of these numbers
 (A) 50 (B) 40
 (C) 20 (D) 30
4. The average of ten numbers is 7. If each number is multiplied by 1, then the average of the new set of numbers is :
 (A) 7 (B) 19
 (C) 82 (D) 84
5. A student was sckrf yo find the arithmetic's mean of the numbers 3,11, 7,9,15,13,8,19,17,21,14 and x, He found the mean to be 12. What should be the number in place of x ?
 (A) 3 (B) 7
 (C) 17 (D) 31
6. The average of 11 results s 60. If the average of first six results is 58 and that of the last six is 63. Find the sixth result.
 (A) 66 (B) 70
 (C) 78 (D) 85
7. The average weight of A,B, C is 45 kg. If the average weight of A and B be 40 kg and that of B and C be 43kg. Find the weight of B.
 (A) 35 (B) 42
 (C) 31 (D) 30

8. The average weight of 10 oarsmen in a boat is increased by 1.8 kg when one of the crew member, whose weighs 53 kg is replaced by a new man . Find the weight of the new man.
 (A) 75 (B) 71
 (C) 68 (D) 80
9. Nine persons went to a hotel for taking their meals. Eight of the spent Rs. 12 each on their meals and the ninth spent Rs.8 more than the average expenditure of all the nine . What was the total money spent by them?
 (A) 115 (B) 117
 (C) 120 (D) 128
10. There were 35 students in a hostel. Due to the admission of 7 new student the expenses of the mess were increased by Rs.42 per day while the average expenditure Per bead diminished by Re 1. What was the original expenditure of the mess ?
 (A) 425 (B) 410
 (C) 420 (D) 430
11. A library has an average of 510 visitors on Sundays and 240 on other days. The average number of visitors per day in a month of 30days beginning with a Sunday is
 (A) 250 (B) 276
 (C) 280 (D) 430
12. The batting average for 40 innings of a cricket player is 50 runs. His highest score exceeds his lowest score by 172 runs. If these two innings are excluded the average of the remaining 38 innings is 48 runs. The highest score of the player is
 (A) 165 runs (B) 170 runs
 (C) 172 runs (D) 174 runs
13. The average of runs of a cricket player of 10 innings was 32. How many runs must in his next innings so as to increase his average of runs by 4 ?
 (A) 2 (B) 4
 (C) 70 (D) 76
14. The average age of 36 students in a group is 14 years. When teacher's age is included to it , the average increases by one. What is the teacher's age in years ?
 (A) 31 (B) 36
 (C) 51 (D) can't be determined
15. The average of a husband and his wife was 23 years at the time of their marriage. After five years they have a one –year old child. The average age of the family now is
 (A) 19 years (B) 23 years
 (C) 28.5 years (D) 29.3 years
16. Ten years ago , average of the ages of a men and his wife was 25 years. Today the average age of these two and their son taken together is again 25 years. What is the age (in years) of the son today ?
(NTSE Stage-11/2008)
 (A) 2 (B) 5
 (C) 8 (D) 10
17. A batsman, by scoring 68 runs in his 20th inning improves his average by 2 runs, His average for all 20 innings is :
 (A) 28 (B) 30
 (C) 32 (D) 34



LINES & ANFLES

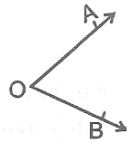
Definition

LINE :

A line has length but no width and no thickness.

ANGLE :

An angle is the union of two non-collinear rays with a common initial point. The common initial point is called the 'vertex' of the angle and two rays are called the 'arms' of the angles.



REMARK :

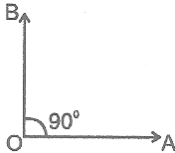
Every angle has a measure and unit of measurement is degree.

One right angle = 90°

$1^\circ = 60'$ (minutes), $1' = 60''$ (Seconds)

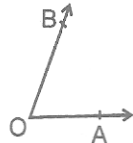
Type of Angles :

(i) Right angle : A angle whose measure is 90° is called a right angle.



(ii) Acute angle: An angle whose measure is less than 90° is called an acute angle.

$$0^\circ < \angle BOA < 90^\circ$$



(iii) Obtuse angle : An angle whose measure is more than 90° but less than 180° is called an obtuse angle.

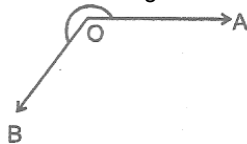
$$90^\circ < \angle AOB < 180^\circ$$



(iv) Straight angle : An angle whose measure is 180° is called a straight angle.

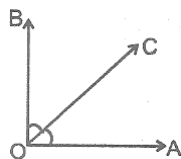


(v) Reflex angle : An angle whose measure is more than 180° is called a reflex angle. $180^\circ < \angle AOB < 360^\circ$

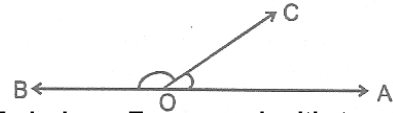


(vi) Complementary : Two angles, the sum of whose measures is 90° are called complementary angles.

$$\angle AOC + \angle BOC = 90^\circ$$

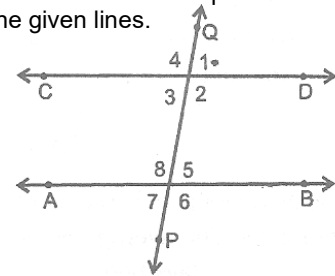


(vii) Supplementary angles : Two angles, the sum of whose measures is 180° , are called the supplementary angles. $\angle AOB + \angle BOC = 180^\circ$



Angles Made by a Transversal with two Parallel Lines

(i) Transversal : A line which intersects two or more given parallel lines at distinct points is called a transversal of the given lines.



(ii) Corresponding angles : Two angles on the same side of a transversal are known as the corresponding angles if both lie either above the two lines or below the two lines, in figure $\angle 1$ & $\angle 5$, $\angle 4$ & $\angle 8$, $\angle 2$ & $\angle 6$, $\angle 3$ & $\angle 7$ are the pairs of corresponding angles.

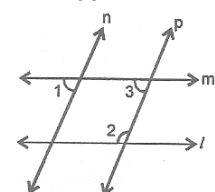
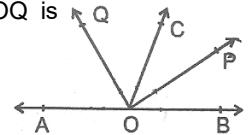
(iii) Alternate interior angles : $\angle 3$ & $\angle 5$, $\angle 2$ & $\angle 8$ are the pairs of alternate interior angles.

(iv) Consecutive interior angles : The pair of interior angles on the same side of the transversal are called pairs of consecutive interior angles.

(v) Vertically opposite angles : in figure, $\angle 1 = \angle 3$, $\angle 2 = \angle 4$, $\angle 5 = \angle 7$ & $\angle 6 = \angle 8$

EXERCISE

- Two supplementary angles are in ratio 4:5 then the angles are .
(A) $80^\circ, 100^\circ$ (B) $70^\circ, 110^\circ$
(C) $80^\circ, 120^\circ$ (D) none
- If an angle differs from its complement by 10° , find the larger angle.
(A) 45° (B) 50°
(C) 95° (D) 60°
- In given figure, OP and OQ bisect $\angle BOC$ and $\angle AOC$ respectively. Then $\angle POQ$ is
(A) 90°
(B) 45°
(C) 135°
(D) 180°
- In given figure if $m \parallel n$ and $\angle 1 = 85^\circ$ then $\angle 2$ is
(A) 95°
(B) 110°
(C) 135°
(D) 85°

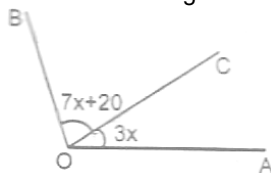


5. An larger angle is 14° more than its completer $\angle ABC$ angle then angle is :
 (A) 38° (B) 52°
 (C) 50° (D) none of these
6. X lies in the interior of $\angle BAC$ if $\angle BAC = 70^\circ$ and $\angle BAX = 42^\circ$ then $\angle XAC = ?$
 (A) 28° (B) 29°
 (C) 27° (D) 30°
7. If the supplement of an angle is three times its complement, then angle is :
 (A) 40° (B) 35°
 (C) 50° (D) 45°
8. Two angles whose measures are a & b are such that $2a - 3b = 60^\circ$, If they form a linear pair then find $\frac{4a}{5b}$

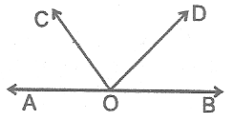
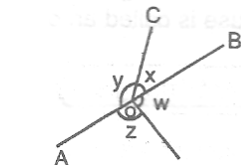
- (A) 0 (B) $\frac{8}{5}$
 (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

9. The supplement of an angle is one third of itself, Then the angle is :
 (A) 135° (B) 45°
 (C) 14° (D) none
10. Two complementary angles are such that two times the measure of one is equal to three times measure of the other . Then the measure of the larger angle is :
 (A) 36° (B) 45°
 (C) 40° (D) 54°
11. Complementary of an angle $36^\circ 40'$ is :
 (A) 54° (B) $54^\circ 20'$
 (C) $53^\circ 20'$ (D) none
12. In given figure, if $\angle BOC = 7 \times X + 20^\circ$ and $\angle COA = 3x$, then the value of x for which AOB becomes a straight line.

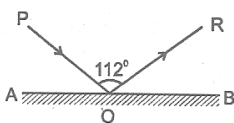
- (A) 15°
 (B) 12°
 (C) 16°
 (D) none



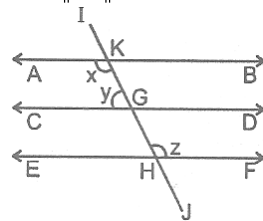
13. In given figure, AOB is a straight line then $x + y$ is equal to :
 (A) $w + z$
 (B) $w - x$
 (C) $x - z$
 (D) none
14. In given figure, AOB is straight line $\angle AOC + \angle BOD = 100^\circ$ then $\angle COD$ is :
 (A) 60°
 (B) 45°
 (C) 80°
 (D) none



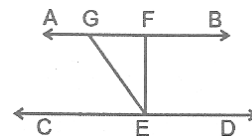
15. In the given figure, AB is a mirror, PO is the incident ray and OR, the reflected ray . If $\angle POR = 112^\circ$ then $\angle POA$
 (A) 34°
 (B) 36°
 (C) 56°
 (D) 68°



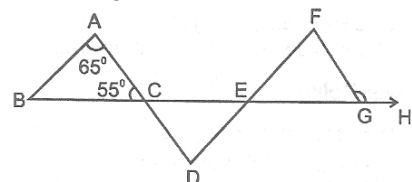
16. In given figure, if $AB \parallel CD \parallel EF$ and $y:z = 3:7$
 (A) 120°
 (B) 126°
 (C) 116°
 (D) none



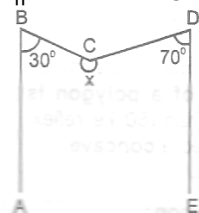
17. In given figure if $AB \parallel CD$, $EF \perp CD$ and $\angle GED = 126^\circ$, then $\angle AGE$ is :
 (A) 126°
 (B) 132°
 (C) 146°
 (D) none



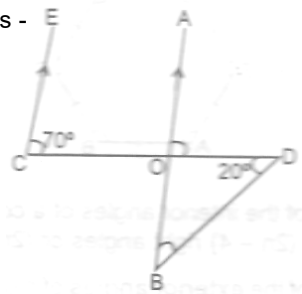
18. In given figure if $AB \parallel DF$, $AD \parallel FG$, $\angle BAC = 65^\circ$, $\angle ACB = 55^\circ$ then $\angle FGH$ is :
 (A) 120°
 (B) 125°
 (C) 115°
 (D) 140°



19. In given figure, $AB \parallel ED$ and $\angle ABC = 30^\circ$, $\angle EDC = 70^\circ$ then x is :
 (A) 240°
 (B) 125°
 (C) 260°
 (D) none



20. In the given Figure, if $EC \parallel AB$, $\angle ECD = 70^\circ$ and $\angle BDO = 20^\circ$, then $\angle OBD$ is -
 (A) 20°
 (B) 50°
 (C) 60°
 (D) 70°



21. In the figure ℓ , parallel to m and AX and AY are transversal. Then the value of the angle $(x + y - z)$ is
 (NSTE Stage - II/2007)
 (A) 110° (B) 80°
 (C) 40 (D) 30°

